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NONVERBAL PROBLEM-SOLVING BEHAVIORS OF BOYS IN GRADES ONE, TWO, AND THREE

by

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A THESIS

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ABSTRACT

The purpose of this study was

a. to catalogue behaviors of boys in first, second and third grades in nonverbal mathematical problem-solving situations,

b. to observe differences in problem-solving behaviors of boys in these three grades,

c. as a result of these observations, to generate, if possible, classes of problem-solving behaviors,

d. to study the effect of the experience of solving problems through the use of manipulative materials on performance with verbally-posed problems.

The sample used in this study consisted of six boys from each grade in an elementary school in the Edmonton Public School System. These boys were selected in a random manner. Ten problems were posed in this study, five based on measurement division and five on partition division. The materials used in these problems were either built or adapted by the investigator. Through a pilot study and from expert opinion, these problems were deemed to satisfy the criteria for "good" problems adopted by the investigator.

Among behaviors observed and recorded on half-inch videotape were partitioning, counting, measuring, classifying, calculating, verbalizing, gestures and contemplating. Of these, counting was the most extensively exhibited behavior. Measurement division appeared to be understood by boys in all three grades. However, partitive division was most systematically performed by the boys in grade three.

The infrequency of this behavior in grade one boys, and the unsystematic use of it by grade two boys appeared to be one of the most significant results of the study.

Verbalizing and gestures were behaviors more characteristic of grade one boys, while calculating appeared mostly with grade three boys.

Behaviors appeared to be broadly categorized as cognitive, affective and motor. The first of these classes included partitioning, counting, measuring, comparing, calculating, miming, observing, contemplating and classifying. The affective behaviors were sighing and fantasizing, and the motor behaviors were gestures and verbalizing.

Verbal problem-solving tests were given to determine the effect of experience with manipulative materials on solving verbal problems. No significant differences were observed between the means of the subjects investigated and those of a random sample of boys taken from the three grades in the school. However, there was a significant difference between the means of the grade one and the grade three subjects.

Problems associated with the conduct of the research and suggestions for future research procedures were included in this report.

The study concluded with several implications for education and some suggestions for further research.

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TABLE OF CONTENTS

LIST OF TABLES

TABLE		PAGE

LIST OF PLATES

CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

> There is no essential difference between the elements recognized by the bodily senses, and those apprehended by pure intellect (Scott, 1896).

Much has been written in the last two decades to suggest that the teaching of problem-solving is as difficult an educational task as it was at the turn of this century. Most textbooks on educational psychology or learning theory contain substantial sections on the topic, for "problem-solving" and "thinking" are envisaged as being highly similar, if not synonomous (Dewey, 1933; Inhelder & Matalon, 1960; Gagne, 1970; Piaget, 1967; Bruner, 1966). However, each educator has his particular theory on the nature of the process involved. Furthermore, research has tended to concentrate in the area of verbal problem-solving (Kilpatrick, 1969).

Psychologists are in general agreement on what constitutes a problem. Dunker (1968) explained that "a problem arises when a living creature has a goal but does not know how this goal is to be reached (p. 28)." Explaining the mechanism of problem-solving, Skemp (1966) wrote, "Problems can be seen as tasks which require accommodation of the pupil's existing schema in greater or lesser degree (p. 76)." The philosopher-educator, Dewey (1933), viewed a problem as "whatever--no matter how slight and commonplace in

character--perplexes and challenges the mind so that it makes belief at all uncertain (p. 12)."

To obtain a statement of what constitutes a mathematical problem, one could consider the definition by Atkin (1966). "A question is a problem when the process or processes by which it is to be solved is not immediately obvious to the person attempting to solve it (p. 49)." These processes would be determined by the functional relationship of the quantities involved in the problem. This relationship determines the mathematical operation or operations required for its solution.

Because the emphasis in recent studies has been on the solution of verbal problems, research has tended to consider particular programs such as Individualized Program Instruction (IPI) (Daly, 1971) and certain strategies for obtaining solutions (Affolter, 1970). The nature of the methods of presentation and the resultant pupil responses meant that the pupil behaviors observed were of a verbal or written form.

According to Ekman and Friesen (1969), nonverbal behavior provides valuable information which may be suppressed if the overt problem-solving behavior is exclusively verbal. These researchers developed a categorization of nonverbal behaviors which identified the relationship between a nonverbal act and its meaning. From this categorization, a subject's thoughts and emotions may be inferred by observing his overt or physical reactions to a particular situation.

By far the most extensive work on the problem-solving processes of children has been done by Piaget. This celebrated

Swiss psychologist conceives of the development of thinking and problem-solving ability as occurring in stages through the child's gradual adaptation to his environment. The dynamic process of adaptation is made up of two components--a child's assimilation or incorporation of objects of the external world into his patterns of behavior, and accommodation or modification of his behavior to fit the environment (Rosenbloom, 1965).

Piaget, with his methode clinicale, searched for affiliations between different levels of overt behavior. According to Inhelder and Matalon (1960),

> these affiliations seem to be ensured by so-called schemata, which, for Piaget, constitute the unities of behavior. A schema is a mode of action elaborated by the subject, a mode of action capable of conservation, of transformation by generalization, and of co-ordination with other schemata (p. 73).

By observing particular genetic affiliations, Piaget was able to construct a theoretical network of cognitive behaviors, the significance of which is becoming universally recognized.

Since the study of human problem-solving processes typically involves observing the way in which a person arrives at a solution to a particular task, a clinical study of children in problem-solving situations of a mathematical nature appears to be significant at this time of curriculum development.

I. PURPOSE OF THE STUDY

Very little has been recorded of investigations regarding

nonverbal problem-solving in elementary school mathematics. Kilpatrick (1969) has urged that more clinical studies be made of individual subjects in problem-solving situations.

One of the perennial problems in elementary mathematics education is the teaching of division. For this reason, problems involving aspects of division were selected as an appropriate area of mathematical significance for this study.

The purpose of this study was to obtain some insight into the problem-solving behavior of a sample of first, second and third grade children, in an attempt to answer the following questions:

a. What specific behaviors are generated when these pupils are exposed to nonverbal problem situations potentially involving the operation, division.

b. How do children in first, second, and third grade differ in problem-solving behavior?

c. Is it possible to generate classes of behavior for nonverbal problem situations potentially involving division, which are typical of children at various levels?

d. What are the levels of attainment of these children in solving division problems when the examples are presented orally to a group?

II. DEFINITIONS

<u>Measurement Division</u>. Measurement division problems are those which require the number of subsets to be determined when a

given set of elements is separated into equivalent subsets.

Partition Division. Partition division problems are those which require the number of elements in each subset to be determined when a set of elements is separated into a given number of equivalent subsets.

Problem-solving Behavior. Those actions generated by a problem situation and directed towards the solving of that problem are defined as problem-solving behaviors.

Level of Solution. A pupil's level of solution will be determined by the overt behavior exhibited, the greater the dependence on the materials available for solving the problem, the lower the level of solution.

Verbal Problem of Arithmetic. A quantitative situation described in words in which a definite question is raised but for which the arithmetical operation is not indicated (Spitzer, 1948, p. 209).

Nonverbal Problem. A quantitative situation represented by a physical model in which a definite question is raised but for which the arithmetical operation is not indicated. The problem is not presented in written form.

III. DESIGN OF THE STUDY

In the absence of any consensus of opinion among researchers of what constitutes a good problem, the characteristics listed by Nelson and Kirkpatrick (1973) as criteria for the construction and

selection of "good" problem situations have been adopted for this study. These characteristics are:

a. The problem should be mathematically significant.

b. The situation in which the problem occurs should involve real objects or obvious simulations of real objects.

c. The situation should capture the interest of the child.

d. The problem should require the child to make moves, or transformations, or modifications with, or in, the materials.

e. Whenever possible, problems should offer opportunities for different levels of solution.

f. The problem should have many physical embodiments.

g. The child should be convinced that he is capable of solving the problem, and should know when he has a solution to it.

Five problem situations were devised, each incorporating the two aspects of division defined earlier, namely, measurement and partition. These situations will be labelled thus:

a. The FENCE situations, problems I and II

b. The ANIMALS situations, problems III and IV

c. The CRANE situations, problems V and VI

d. The FERRY situations, problems VII and VIII

e. The SKYTRAM situations, problems IX and X.

Of these problems, a., b., and e. were original problems created by the investigator who also constructed the manipulative materials for their solution. The remaining problems were devised and described

by Nelson and Kirkpatrick (1973).

Population

An elementary school within the Edmonton Public School System was assigned, and comprised two first grade, two second grade, a combined first and second grade, and two third grade classes. Eighteen boys, six from each grade, were selected in a random manner by the principal by choosing every third name on the class lists.

The Setting

The problems were presented individually to each child, who was invited to use whatever materials he needed to arrive at a solution. When the child indicated he was satisfied he had solved the problem presented, he was then introduced to the next problem. All overt behavior and verbalization which occurred while the child was engaged in solving the problem were recorded on videotape. Analysis of the data so recorded will appear in the fourth chapter of this thesis.

When all videotaping had been completed, an oral test was administered to all children in the three grades. This test consisted of ten questions, five on measurement division and five on partitive division, given alternately. These questions were composed by the investigator and were designed to conform with the criteria listed to characterize "good" problems referred to at the beginning of this

section.

IV. ASSUMPTIONS

The following assumptions were formulated in relation to the study:

a. It was assumed that the problems would generate a wide variety of behaviors on the part of each child, these behaviors being related to different levels of solution of the problems.

b. It was assumed that each child would find each problem interesting, and would proceed with the problem until a solution satisfactory to him was reached.

V. LIMITATIONS

The study had the following limitations:

a. Many variables which may affect problem-solving behaviors, such as computational skills, curiosity, persistence, previous success with problem-solving, and attitude to arithmetic, were not controlled.

b. The behaviors observed may be characteristic only of the sample of boys used in this investigation.

c. The number of problem-solving situations was restricted to five involving measurement division and five involving partitive division.

d. The suitability of each problem was based on the criteria to

which earlier reference was made.

VI. SIGNIFICANCE OF THE STUDY

The study is considered important for the following reasons:

a. Researchers in the field of problem-solving (Kilpatrick, 1969; Kleinmuntz, 1966) have indicated the need for further research into problem-solving behaviors, and specifically, for the investigation of individual case studies.

b. The study may have curricular implications regarding suitably structured materials for fostering problem-solving skills in the elementary school classroom.

c. There is a need for suitable relevant problems which have intrinsic interest to young children. The problems in this study relate what is happening in the real world to its mathematical symbolic representation in the classroom. In this way the problems devised for this study may act as a guide to the profession for the construction of similar problems. As Newell and Simon (1972) comment,

> a theory of the psychology of problem-solving requires not only good task analyses but also an inventory of possible problem-solving mechanisms from which one can surmise what actual mechanisms are being used by humans (p. 6).

VII. THE OUTLINE OF THE REPORT

In Chapter I the problem is introduced. A detailed review of the related literature and research is given in Chapter II. The design of the investigation is described in Chapter III together with the methods and materials used in the study and the method of analysis used to answer the questions proposed. In Chapter IV the data are analysed. Finally, a summary of the study and reports on conclusions and implications are given in Chapter V. Some suggestions for further research are also included.

CHAPTER II

A REVIEW OF RELATED LITERATURE AND RESEARCH

Because much of our attention is directed towards thinking and solving problems of a practical nature, it is understandable that psychologists and educators should associate these two processes. Certainly much has been published in recent decades to suggest that both thinking and problem solving are essential aspects in the study of concept development. Suydam (1969) reported that problem solving has become the most popular topic for research in elementary school mathematics. At the same time a diversity of opinion based on research, exists as to the nature of the thinking process (Berlyne, 1970). It would appear appropriate, therefore, in this review of the literature associated with problem solving, that space be devoted to some consideration of the thinking process. This will be followed by an examination of learning theory, cognitive development, theories of problem solving, problem solving methods, and finally a brief survey of problem solving research with emphasis on nonverbal behavioral research.

I. THINKING

The thinking process has attracted the attention of philosophers in all ages, and in recent times has taxed the energies of psycholo-

gists and educators. However, a definition of thinking which would satisfy the different approaches found in the literature would be rather complex. Berlyne (1970) has expressed it thus: "Thought has sometimes been conceived so broadly that it encompasses virtually all conscious or internal psychological processes (p. 939)." A more simplistic and extreme view is that of Skinner (1957), that "thought is simply behavior--verbal or nonverbal, covert or overt (p. 449)."

The uniqueness of human thought and its intimate connection with volition were emphasised by the rationalist-spiritualist school of the seventeenth to nineteenth century. This European approach viewed behavior as determined by cognition, understanding and judgement. This was in contrast with the associationistic tradition in Great Britain which regarded thought as being derived from sensation and action (Berlyne, 1970).

The present century has witnessed a similar contrast in psychological thought. The Gestalt school, characterised by Wertheimer, considered behavior to be governed, in the main, by "representations, models or images" of internal reality (Berlyne, 1970, p. 941), that is to say, by factors related to thinking. On the other hand, the neobehaviorists of the stimulus-response (S-R) school did not assume that representational processes intervened between stimulus and response, but simply considered thoughts as covert or internal responses derived from overt motor responses. Contemporary S-R psychology is more concerned with associations between classes of stimulus situations and classes of behavior than with a single stimulus and its response. In the last decade great

progress has been made by Newell and Simon (1972), who, with an information processing theory, developed computer analogues and analysed thought in terms of storing and processing information.

Bartlett (1958) treated thinking as a complex and high level kind of skill acquired by well-informed practice. He wrote ". . .we should be content to regard thinking as an extension of evidence, in line with that evidence and in such a manner as to fill up gaps in the evidence (p. 20)." These gaps were to be filled by a series of interconnected and well articulated steps. Bartlett went on to discuss four categories of thinking.

a. Closed systems, in which some information was given and the subject was required to provide additional facts by interpolation or extrapolation.

b. Experimental, in which the subject is able to expand a structure not yet complete, by visualizing new relationships and by using tools which may be available or devised.

c. Everyday, in which the subject generalizes, decides, and compromises on a subjective basis, and assigns verity to conclusions unwarranted by facts.

d. Artistic, an independent thinking style not restricted by convention, but based on personal judgement and satisfaction.

Bartlett appeared to be more concerned with the method employed than with the interaction between the person and the environment.

II. LEARNING

The application of thinking to learning and problem solving has been the subject of much research (Johnson, 1961; Mednick, 1964; Berlyne, 1965; Kagan and Haveman, 1968; Olson, 1970; Torrance, 1970). The following definition does draw attention to the characteristics of learning.

> Learning is the process by which an activity originates or is changed through reacting to an encountered situation, provided that the characteristics of the change in activity cannot be explained on the basis of native response tendencies, motivation, or temporary states of the organism (eg. fatigue, drugs, etc.) (Hilgard and Bower, 1966, p. 2).

Thus modification of behavior is a result of learning. By reinforcement through practice, the permanency of this change may be increased. Although the results of learning may be clearly evident, learning itself is not directly observable.

Theories of learning are more diverse than theories of thinking, ranging as they do through

a. Thorndike's Connectionism,

b. Pavlov's Classical Conditioning,

c. Guthrie's Contiguous Conditions,

d. Skinner's Operant Conditions,

e. Hull's Systematic Behaviour Theory,

f. Tolman's Sign (Behaviorism) Learning,

g. Gestalt Theory,

h. Information Processing Theories of Behavior.

It is not the intention of the investigator to appraise the postulates of these various theories, but simply to indicate that they cover a wide range of psychological thought. However, certain of these theories will appear in later sections, where their relevance to problem-solving theories seems important.

III. COGNITIVE DEVELOPMENT

Before considering methods of problem solving which have evolved, it is pertinent to review the contributions of two theories to present day educational practice. The first of these is based on the work of Piaget (1952) who emphasizes motivational processes as well as experience, social transmission and equilibration in the transfer from stage to stage. The second has Gagné (1965a, 1965b, 1965c, 1966a, 1968) as its major exponent, and emphasizes learning as a hierarchical process.

Maturational Processes

Representative of a theory with a maturational basis is what Bruner (1966b) has called instrumental conceptualism. By this term Bruner means that knowledge is bounded by internal characteristics of the organism and by external characteristics of the environment within which the organism grows.

> . . .our knowledge of the world is based on a constructed model of reality, a model that can only partially and intermittently be tested against input. . . our models develop as a function of the uses to which they have been put first by the culture

> and then by any of its members who must bend knowledge to their own uses (p. 320).

A notion basic to this theory of cognitive development is that of representation, namely, the ways in which the organism's experience with or reaction to the environment are encoded and stored. There are, according to Bruner, three modes of representation, action (enactive), image (ikonic), and symbol (symbolic representation). Although these modes are flexible in application, there is also an obvious hierarchy by virtue of their sequence in the activities of a child. It could be expected that the enactive mode would be encountered first, followed by the ikonic and symbolic modes. Once all modes have been experienced by a child, he can choose whichever mode he prefers to use to interpret his particular environment, and hence their flexibility.

From the viewpoint of instrumental conceptualism, one could expect cognitive development to be characterised by plateaus and abrupt changes observable only through logitudinal studies. According to Bruner (1966a), "the heart of the educational process consists of providing aids and dialogues for translating experience into more powerful systems of notation and ordering (p. 21)." Hence the importance of developing these modes of representation.

Piaget (1968) has provided the most thoroughgoing account of the various aspects of the development of representational functions. Primitive signs of symbolization, showing the ability to recognise properties of an object, occur during the first two years of life. This period has been designated by Piaget as the

sensorimotor stage of cognitive development. A major turning point for the child occurs at the end of this stage when behavior comes under the control of words, images, and other symbolic processes, and the child proceeds through the preconceptual stage (2 to 4 years approximately). During the intuitive stage (4 to 7 years) concepts of class membership and class inclusion become meaningful.

The stage of concrete operations begins at about the age of 7, when thinking makes use of operations as "actions which are internalised, reversible and coordinated in systems characterised by laws which apply to the system as a whole (Piaget, 1953, p. 23)." From about age 11 to 15 years, the child experiences the formal operations stage, during which he becomes equipped to cope with advanced logical, mathematical, and scientific thinking. These stages delineated by Piaget illustrate the plateaus which characterized the theory of instrumental conceptualization mentioned earlier.

Hierarchical Processes

Gagné is representative of a number of psychologists such as Guilford, Osgood, and Skinner, who emphasize the dependence of learning on cognitive development and education rather than on maturation. Gagné (1970) asserts that acquisition processes are hierarchically organised, and distinguishes eight varieties of learning: Signal learning (conditional response), Stimulus-Response learning (motor acts), Chaining (S-R connections), Verbal associations (or chains), Discrimination learning (multiple S-R

connections), Concept-learning, Rule learning and the highest order learning, namely Problem solving. Each of these is a prerequisite for its successor, showing that the more complex forms of learning are dependent on prior, simpler forms.

> If growth is the dominant theme, educational events are designed to wait until the child is ready for learning. In contrast, if learning is a dominant emphasis, the years are to be filled with systematically planned events of learning and there is virtually no waiting except for the time required to bring about such changes (Gagné, 1968, p. 171).

Both theories are in agreement with the fact that learning occurs through a modification of behavior, taking into account the reciprocal action between the organism and the environment.

IV. THEORIES OF PROBLEM SOLVING

It seems logical that a theory of problem solving should evolve from a theory of thinking. Consequently, names now familiar to the reader will be expected to appear.

At the beginning of the twentieth century Thorndike conducted a comprehensive study of problem solving with cats and other animals. From these experiements he derived his principle of learning, "whatever behavior is rewarded is 'stamped in' and whatever behavior is not rewarded or is punished is 'stamped out' (Scheerer, 1966, p. 147)." Although half a century has passed, this principle is still accepted and is central to contemporary S-R psychology typified by Skinner (1966).

> Problem solving is concerned with the relations which prevail among three terms, a stimulus,

> a response, and a reinforcing consequence. . . when a response occurs and is reinforced, the probability that it will occur again in the presence of stimuli is increased. This is simple operant conditioning (p. 226).

This typifies the operant conditioning techniques of the Skinnerian school of thought on problem solving. However it does not seem to allow for the adaptation which must necessarily take place when the problem solver finds himself in a new situation.

The Gestaltist approach to problem solving, represented by Maier (1970) and Dunker (1945), emphasises

> the tendency of the mind to organise and integrate and to perceive situations, including problems, as total structures. They argued that more is involved in problem-solving than a sequence of stimuli and responses. . . The insight that leads to a solution stems from this perception of the requirements of a problem (Scheerer, 1966, p. 147).

In contrast to Thorndike's dependence on the occurrence of a suitable behavior for a solution to occur, the Gestaltists believe that

> productive thinking is not accidental success or the mere application of bits of the past experience. The problem has a structure of its own that points the way to its solution. Only within this total framework, or context, does the problem solver draw selectively on relevant knowledge (p. 149).

Dunker (1945) demonstrated "that the whole process, from the original setting of the problem to the final solution, appears as a series of more or less concrete proposals (p. 33)." It can be seen that he perceived problem solving as a process whereby the problem solver arrives at a final solution by way of a number of solutions to intermediate problems.

Gagné (1970) agrees with the Gestaltist ideas of structure, insight, mental set, and the need for a store of intellectual skills. However his hierarchical concept of learning, mentioned earlier, envisages problem solving as the end point in a sequence commencing with specific responses, chains, discriminations, concepts, and rules. He wrote:

> problem solving may be viewed as a process by which the learner discovers a combination of previously learned rules that he can apply to achieve a solution for a novel problem situation (p. 214).

Later he adds, "what emerges from the problem solving is a higher-order rule, which therefore becomes a part of the individual's repertory (p. 216)."

The highly structured nature of his learning theory makes Gagné, along with Polya (1967), a forerunner of the information processing theorists such as Newell, Simon and Shaw (1958). The heuristic approach of Polya, together with a highly structured hierarchical learning theory, led Newell and his associates to develop an information processing program which drew on the details about problem solving from everyday experiences and research. Working on the elements of a theory of human problem-solving, these researchers programmed a computer to perform problem-solving tasks in the same manner as a human. This resulted in the acquisition of a deeper insight into problem-solving behavior.

> From formal logic, information processing inherited an emphasis upon syntax, upon games with arbitrarily defined rules, played with symbol tokens. The meaning of symbols, what they denote, play a muted role in this scheme. . . Even without external reference, complex processes operating on complex memory structures require addressing

> schemes so that one symbol can denote another. Semantics . . . has reemerged as a central concern in information processing. (Newell and Simon, 1972, p. 889).

V. PROBLEM SOLVING METHODS

Hodnett (1955) has described a problem as essentially "a state of disorder" and the solving of problems as "the search for order, for the overcoming of disorder (p. 3)." Gagné defined problem solving as "an inferred change in human capability resulting in the acquisition of a generalizable rule novel to the individual, and applicable to the solution of a class of problems (Kleinmuntz, 1966, p. 132)."

According to Dewey (1933), reflective thinking begins when there is a doubt or perplexity that leads to a search for material that will resolve the doubt and dispose of the perplexity. This activity produces a resolved, unified situation at its conclusion. Dewey proposed a five phase development of problem solving, namely:

1. Suggestions, in which the mind anticipates possible solutions. This phase encourages thinking by inhibiting direct action.
2. Definition of the problem or intellectualization. Here the problem-solver assesses the conditions that caused the perplexity, locates and defines the problem.
3. Formulation of hypotheses to initiate and guide the search for relevant material.
4. Mental elaboration of the idea, which is simply mental

testing of the hypothesis.

5. Testing the hypothesis by overt action.

These five phases of thought may occur in this order, and some phases may be passed over quietly. A solution may occur at any intermediate stage, which, in turn, may become so complex that subphases develop.

The problem-solving process developed by Dewey has been the basis of a common method of problem solving in schools.

> The traditional approach to the solving of word problems in the elementary school grades has been characterised as a "wanted--given" procedure--the child is taught to ask himself, What is wanted? and What is given? and then to perform the appropriate operations on the data to yield values for the unknowns. (Kilpatrick, 1969, p. 530).

Following the Dewey model, recent problem-solving frameworks have generally consisted of several phases, ranging in number from three to eight, and bear many similarities to each other.

Gray (1935) suggested a five phase model:

a. Sensitivity to problems.

b. Knowledge of problem conditions.

c. Suggested solutions or hypotheses.

d. Subjective evaluation.

e. Conclusion or generalization.

Osborn (1963) included incubation in his model:

a. Orientation.

b. Preparation.

c. Analysis.

d. Hypothesis.

e. Incubation.

f. Synthesis.

g. Verification.

Wallas (1970) lists what is sometimes referred to as the scientific method:

a. Preparation.

b. Incubation.

c. Illumination.

d. Verification.

Hadamard (1945) referred to the efficacy of a period of incubation for the illumination of the solution of a particularly difficult problem and cited instances which included such outstanding men as Poincare, Helmholtz and Mozart.

> Thus Gauss, referring to an arithmetical theorem which he had unsuccessfully tried to prove for years, writes: "Finally, two days ago, I succeeded, not on account of my painful efforts, but by the grace of God. Like a sudden flash of lightning the riddle happened to be solved. I myself cannot say what was the conducting thread which connected what I previously knew with what made my success possible (p. 15).

Polya (1957) advocated four steps:

a. Understand the problem.

b. Devise a plan which will guide the solution and connect the data to the unknown.

c. Carry out the plan of the solution, checking each step.

d. Verify the completed solution, reviewing, checking, discussing.

VI. PROBLEM SOLVING AND RELATED VARIABLES

As previously mentioned, Suydam (1971), along with Riedesel (1969) and Stievater (1971) has reported the abundance of research on problem-solving ability, problem-solving content and structure, and problem-solving processes. It should be noted that investigations have been associated almost without exception with routine verbal problem-solving.

Problem-solving and Mathematics Achievement

Steffe (1968) investigated the relationship between conservation of numerousness and problem-solving abilities of first-grade children. He made a random selection of 132 grade one children and administered three tests; namely a test of conservation of numerousness, a test of eighteen addition problems, and an addition facts test. Of the eighteen problems, nine involved a transformation, while nine did not. Again, six of the problems had accompanying physical aids, six had accompanying pictorial aids, and six had no accompanying aids. On the basis of the conservation of numerousness test, the children were stratified into four levels. Those in the lowest level performed significantly less well in the two addition tests, while problems with no accompanying aids were significantly more difficult to solve for children in all levels of conservation of numerousness than those problems with accompanying aids. Furthermore, those problems

requiring a transformation were significantly easier for the children to solve at all levels of conservation of numerousness. Working with a group of 492 subjects, Law (1972) investigated the relationship between problem solving ability and age, mathematical competence and overall academic ability. He found that ability in problem solving may be related more closely to intelligence and general academic strength than to mathematical ability as demonstrated by orthodox school examinations in mathematics. However Alexander (1960), Butler (1956), Hansen (1944) and Chase (1960) all reported that arithmetic computational ability was significantly related to the ability to solve verbal problems in arithmetic.

Problem Solving and Reading Ability

When investigating reading skills measured by subtests from two standardised tests, Treacy (1944) found significant differences between high and low achievers in verbal problem-solving ability. In particular, Chase (1960) found that reading to note details was significantly related to ability to solve verbal arithmetic problems. Engelhart (1932), and Harootunian and Tate (1960) also reported that general reading ability was associated with arithmetic problem-solving ability.

Other reading variables which have appeared to be positively correlated with either mathematics achievement or problem-solving include vocabulary, retention of details, perception of relationships, integration of dispersed ideas, reading comprehension, and rate of comprehension (Dodson, 1972).

Multivariate Investigations

Dienes (1960) accepts Piaget's theory of cognitive growth. His theory of mathematics-learning is based on four principles, namely: Dynamic Principle, Constructivity Principle, Mathematical Variability Principle, Perceptual Variability Principle or Multiple Embodiment Principle. Taking the form of problem-solving situations, games based on these principles have been investigated in England, Italy, Australia and Canada (Dienes, 1960, 1963, 1965, 1967). However, Dienes is primarily concerned with the child's ability to discover abstract mathematical structures.

In comparing the problem solving abilities of students following traditional and IPI programs, Daly (1971) arrived at the following conclusions:

a. Socio-economic status does not appear too closely related to problem solving abilities.

b. Problem-solving ability is directly related to problem-solving achievement and intelligence.

c. A subject's intelligence seems to be a better predictor of achievement, on a test composed of difficult and unfamiliar questions, than knowledge of the mathematics programme to which the subject has been exposed.

The effectiveness of an inquiry approach for the development of efficient strategies of problem solving within a grade six arithmetic program, was investigated by Affolter (1970). She also examined the relationships among language achievement, tentative

thinking and problem solving. The study, which involved a sample of 80 students showed that "...strategies (of problem solving) do exist and that they are related to a cycle of mental activity during which a child first of all senses or grasps the significance of a problem, seeks difficult ways of predicting or hypothesizing while searching for a solution, and attempts to verify his predictions (p. 83)." Affolter also found that there was no significant difference between the sexes in problem solving ability as measured by the Strategies of Problem Solving Test. This claim is supported by Cleveland and Bosworth (1967), although Neill (1966) and Sheehan (1968) observed that boys were superior to girls in problem-solving ability. Results concerning this particular variable seem to be inconclusive, and could be influenced by the types of problems involved in the investigations.

VII. PROBLEM-SOLVING PROCESSES

The way in which a problem solver arrives at a solution to a problem has been difficult to investigate. Dunker (1945) used the "thinking aloud" technique to study how complex mathematical problems can be solved. Gagné and Smith (1962) explored the effects of verbalizing on problem solving performance. Studies by Katona (1940) and Haslerud and Meyers (1958) suggested that non-verbalizing was superior to verbalizing by subjects when solving mathematical problems. Gagné and Smith (1962) used 28 boys aged 14-15 years with I.Q.s well above 110, to determine the effects on problem solving

performance of the requirement that subjects verbalize during practice and of instructions to find a general principle and state it verbally.

The results appear to indicate that requiring subjects to verbalize during practice has the effect of making them think of new reasons for their moves, and thus facilitates both the discovery of general principles, and their employment in solving successive problems (p. 16).

The role of insight in problem-solving and the phenomenon of fixation or set was investigated. Luchins (1942) described a set or "Einstellung" as a predisposition to one type of motor or conscious act in which a person allows his previous experience to prevent him from seeing a new problem on its own merits.

> The relevance of an Einstellung is that such a set materially affects the search model, narrows the field of search, inhibits the perception of certain relationships, and blocks certain hypotheses. The results of this is that not all relevant thought material is made available (Henderson & Pingry, 1953, p. 243).

The effects on creative thinking by a method called "brainstorming" has been investigated by Parnes and Meadow (1959), who instructed a subject to attempt to solve problems by recording all tentative solutions which occurred to him. Any evaluation of these solutions was postponed to some later time.

The investigators used 52 university undergraduates with major findings as follows:

a. Significantly more good ideas were produced under brainstorming instructions than under nonbrainstorming

instructions.

b. The subjects trained in a creative problem solving course which emphasized brainstorming produced a significantly greater number of good quality ideas than did the untrained students.

A study by Bloom and Broder (1950) categorised differences between successful and unsuccessful problem-solvers according to:

a. comprehension of the nature of the problem. The successful students immediately recognised some clue enabling them to construct a solution. Misinterpretation and missed clues hindered the unsuccessful.

b. Comprehension of the ideas contained in the problem. The major difference between the two groups of students was the ability to use the knowledge given to solve the problems.

c. The style of problem-solving behavior. The successful students generally thought aloud, and had the ability to dissect the problems as well as to analyse and simplify the component parts. Inability to analyse the problems and frequent guesswork characterised the low achievers.

d. Attitude towards problem-solving. The successful students demonstrated more confidence in approaching the problem.

VIII. NONVERBAL BEHAVIOR AND PROBLEM SOLVING

The prime concern of the investigation was the nonverbal

behavior of subjects in problem-solving situations. In their discussion of the role of problem solving in elementary school arithmetic Cohen and Johnson (1967) maintained that:

> A true problem (often called a "good" problem) in mathematics can be thought of as a novel situation for the individual who is called upon to solve it. The path to the goal is blocked and the individual's fixed patterns of behavior or habitual responses are not sufficient for removing the block. Hence, deliberation must take place. In this deliberation we can note many different kinds of behaviors that might be exhibited by the problem solver. These behaviors can be described in such terms as the following: observing, exploring, decision making, organising, recognising, remembering, supplementing, regrouping, isolating, combining, diagramming, guessing, classifying, formulating, generalising, verifying, and applying. (p. 261).

These descriptions of behavior in general have particular significance for nonverbal behavior in problem-solving situations. Yet a search of the literature revealed a minimal amount of research devoted to this topic.

> Much has been said lately about the need for large-scale complex studies in mathematics education, but the researcher. . . who chooses to investigate problem solving in mathematics is probably best advised to undertake clinical studies of individual subjects. . . because our ignorance in this area demands clinical studies as precursors to larger efforts (Kilpatrick, 1969, p. 531).

Some research in the area of nonverbal behavior of young child has been carried out. These have been basically clinical studies and the field of investigation has been interpersonal and intrapersonal relationships as evidenced by non-verbal behavior in group situations (Birdwhistell, 1970; Turner, 1972; 1973). Turner (1972) found that nonverbal communication cues emitted by a child

in an early childhood setting was a significant part of his total communication. Furthermore, the quantity of nonverbal communication was affected by the situation, and in self-selected, self directed activities the communication was more symbolic and attentive than during teacher initiated, teacher directed activities.

Ekman and Friesen (1969) developed categories of non-verbal behavior by a detailed study of the body movements of individuals, relating these to physiological and emotional characteristics. The categories listed by these researchers were:

a. Emblems -- the most easily understood non-verbal behavior, being acts or gestures which are in common usage, and designed to repeat, substitute or contradict some part of the concommitant verbal behavior.

b. Illustrations -- or movements directly tied to speech and characterised by

(i) batons (which emphasise rhythmic tempo),

(ii) ideographs (movements tracing a path of thought),

(iii) deictic movements (pointing to a present object),

(iv) spatial movements (depicting a spatial relationship),

(v) kinetographs (movements depicting body actions),

(vi) pictographs (a picture drawn of the referent).

c. Affect displays -- the primary source is the face. A partial list of primary affects includes happiness, surprise, fear, sadness, anger, disgust and interest.

d. Regulators -- these do not carry message content in themselves, but tend to regulate the pace of conversation

or action.

f. <u>Adaptors</u> -- such as facial behaviors, for example, the biting of a lip and the closing of the eyes, which the authors state are very difficult to describe or to interpret.

In addition, a skilled observer was required to differentiate between behaviors identified as

(i) informative - which have a shared decoded meaning,

(ii) communicative - intended by the sender to transmit a message,

(iii) idiosyncratic - peculiar to an individual, and

(iv) interactive - designed by the sender to modify or influence the receiving interaction.

Although presenting a fascinating spectrum of possible applications to the study of problem-solving behaviors, the work of Ekman and Friesen was concentrated on research in the affective domain, and has only a limited bearing in the present study. However, it illustrated techniques which, in the future, may become useful for further research in the field of nonverbal problem-solving behavior. Both are involved in the reflex and experiential origins of the relationship between stimulus and the nonverbal behavior due to the motor and cognitive systems of the subject.

SUMMARY

The first three sections of this chapter were devoted to a

consideration of thinking, learning and cognitive development. Their relationship to problem solving ability was established by reference to psychological and educational literature.

In the next two sections the topics Problem-Solving, Methods and Theories of Problem-Solving were traced.

If a synthesis could be made of the methods found in the literature to describe the act of problem solving they would appear as (i) a sensing of the situation, (ii) a predicting of the solution, and (iii) a verification of the solution. Other areas of research examined included research on Problem Solving abilities, content and structure, and processes were also reviewed.

The final section brought into focus some aspects of non-verbal behavior and the terminology for identifying these. Although perhaps too sophisticated for this present research project, no doubt the techniques reported will have greater significance as more information of the type gathered in this study becomes available.

CHAPTER III

DESIGN OF THE INVESTIGATION

The purpose of this investigation was:

a. to catalogue behaviors of boys of first, second and third grades in nonverbal mathematical problem-solving situations,

b. to observe differences in problem-solving behaviors of boys in these three grades,

c. as a result of these observations, to generate, if possible, classes of problem-solving behaviors,

d. to study the effect of the experience of solving problems through the use of manipulative materials on performance with verbally-posed problems.

This chapter provides details of the subjects selected for the study, the problems which were presented, the models which were available for use in solving the problems, and the methods used for analysing the data collected.

I. THE POPULATION

Because of the exploratory nature of the study, a request was made to the Director of Research of the Edmonton Public School Board for a school in which boys from the three grades of the lower

division could be selected.

The elementary school assigned contained only junior division classes, two grade one classes, two grade two classes, a combined grade one and grade two class, and two grade three classes.

II. THE SAMPLE

The children used in this study were selected by the principal from the class lists in a manner described on page 7. A child's eligibility for selection was determined by the following criteria:

a. only boys would be chosen from the class lists,

b. boys who were repeating a grade would not be selected.

A total of twenty one boys were chosen, six from each of the three grades, with three alternates in case illness caused a boy to be absent on the day he was scheduled for testing.

The ages of the boys ranged from 76 months to 111 months. These chronological data, together with available ability measures, were obtained from the school record cards and are listed in Tables 1, 2 and 3. The Metropolitan Readiness Test, published by Harcourt, Brace and World, Inc., claims to assess children's readiness for formal grade one learning in linguistic maturity, perceptual abilities, muscular coordination and motor skills, number and letter knowledge, ability to follow directions and attention span. Table 1 lists the

age in months, the number score obtained (maximum = 26), the total score in the list, the percentile range, the letter rating and readiness status for each grade one boy in the order taken for videotaping. Thus "13" means that the boy was in grade one and was third in the recording of behaviors. Letter rating and readiness status are general descriptions of the score ranges which contain the total scores listed, and are an indication of a pupil's prospects for success in grade one.

TABLE 1

METROPOLITAN READINESS TEST SCORES FOR GRADE ONE SUBJECTS

Subject	Age (Months)	Number	Total	Percentile Rank	Letter Rating	Readiness Status
11	80	18	88	99	A	Superior
12	76	*	*	*	*	*
13	76	6	42	27	D	Low normal
14	79	11	62	65	C	Average
15	86	14	67	75	B	High normal
16	82	16	75	89	B	High normal

*Not available.

Table 2 records the Gates MacGinitie A Scores for the boys selected from grade two, together with their ages. The Metropolitan Readiness Test A Scores were not available from these record cards. The test consists of two parts, a vocabulary test sampling a child's ability to recognize or analyse isolated words given in increasing difficulty, and a comprehension test which measures a child's ability to read and understand whole sentences and paragraphs. The scores in each test indicate the grade level equivalent to the raw score obtained in the test.

TABLE 2

GATES-MACGINITIE A SCORE FOR GRADE TWO SUBJECTS

Subject	Age (months)	V (voc)	C (comp)
21	93	1.7	1.7
22	93	3.2	2.7
23	88	3.2	3.2
24	95	3.2	3.6
25	90	2.7	1.7
26	96	3.2	3.2

The Gates MacGinitie scores and the Lorge Thorndike A scores from the boys in grade three were available and are listed in Table 3.

The investigation was primarily designed to catalogue and classify pupil behaviors in nonverbal problem-solving situations. For this reason it was not considered necessary to administer any test of mathematical or general ability to the sample population. The unavailability of an ability score common to all three grades was not considered to be a serious deficiency.

TABLE 3

AGE, READING AND I.Q. SCORES FOR GRADE THREE SUBJECTS

Subject	Age (Months)	Gates MacGinitie Score (voc)	(comp)	Lorge-Thorndike-A
31	101	2.6	3.7	116
32	107	4.6	4.3	120
33	112	4.9	4.9	129
34	104	5.2	4.9	118
35	104	2.3	1.9	97
36	106	1.9	1.6	92

III. THE PROBLEMS

There were five problem-solving situations. In order to provide instruments for observing children's reactions to measurement and partition types of division problems, each situation was used to present two problems. Each of these problems, described below, was presented to each boy, and his behaviors while solving the problem were recorded. To identify these problems they have been labelled as follows:

a. The FENCE situations, Problems I and II

b. The ANIMALS situations, Problems III and IV

c. The CRANE situations, Problems V and VI

d. The FERRY situations, Problems VII and VIII

e. The SKYTRAM situations, Problems IX and X.

The odd-numbered problems were based on partition division, and the even-numbered problems were based on measurement division.

A detailed description of each problem is now reported which includes a photograph of the way it was set up. After each description, a statement of the problem as it was posed to each boy is then presented.

Problem I

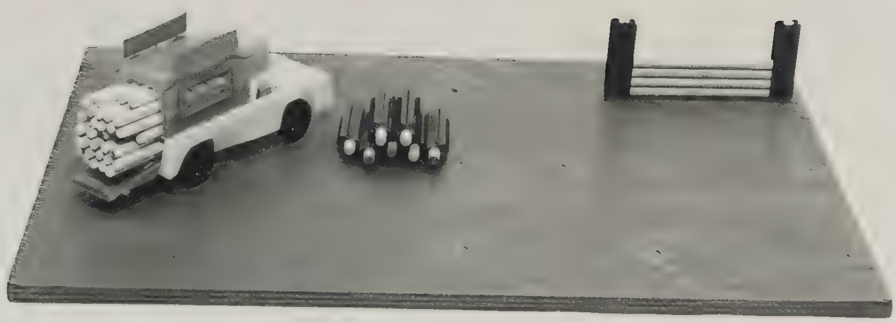

PLATE I

Materials

A base board 90 cm x 60 cm x 2 cm had a three by three array of holes, drilled to accommodate wood dowelling with diameter 1.2 cm. Nine posts were provided, which were made from the dowelling and plastic track. A model farmyard truck contained 32 poles. A section of fencing, consisting of two posts and four poles, was set up as an example and completed the equipment for this problem.

Statement of Problem

"Show me how you would find out how many fences just like that one you can make with that truckload of poles."

(This is an example of measurement division.)

Problem II

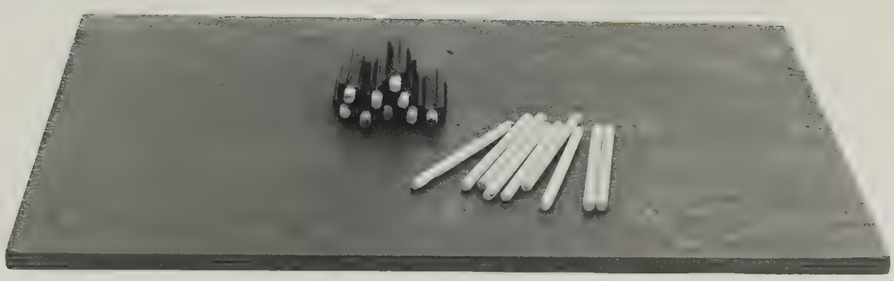

PLATE II

Materials

The baseboard used in problem I was supplied together with the nine posts and nine poles.

Statement of Problem

A farmer wants to make three fences all the same with this lot of poles. "Show me how you would find out how many poles will be in each fence."

(This is an example of partition division.)

Problem III

PLATE III

Materials

The baseboard and posts used in the previous problems were supplied to the student. Wall slabs made of cardboard 19 cm x 5 cm were also available. A set of farmyard animals, which comprised four horses, four sheep, four pigs and four ducks, was placed on the baseboard.

Statement of Problem

A farmer wants to put four animals in each pen. "Show me how you would find out how many pens he needs for these animals."

(This is an example of measurement division.)

Problem IV

PLATE IV

Materials

The baseboard, posts and cardboard slabs used in problem III were available together with a set of plaster zoo animals comprising three each of apes, bears, buffalo, camels, elephants, giraffes, hippopotamuses, and rhinoceroses.

Statement of Problem

A zoo-keeper wants to put these animals into three cages with the same number of animals in each cage. "Show me how you would find out how many animals go in each cage."

(This is an example of partition division.)

Problem V

PLATE V

Materials

A cubical box 25 cm x 25 cm x 30 cm representing a building was surmounted by a crane. Twenty cubes of wood of side 2.5 cm were at the bottom of the building.

Statement of Problem

A crane driver brings these blocks up to the roof four at a time. "Show me how you would find out how many loads of blocks he brings up to the roof."

(This is an example of measurement division.)

Problem VI

PLATE VI

Materials

The box and crane as used in the previous problem, together with 15 wood cubes.

Statement of Problem

The cranedriver wants to bring this group of blocks up to the roof in three equal loads (three loads all the same). "Show me how you would find out how many blocks he puts on each load."

(This is an example of partition division.)

Problem VII

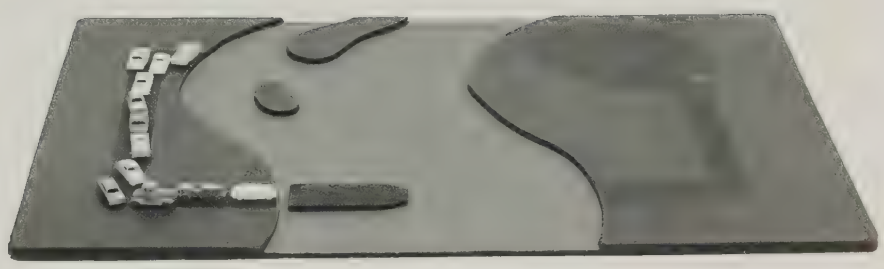

PLATE VII

Materials

The baseboard 90 cm x 60 cm had a river, road and car-park complex painted on it. Twelve plastic toy cars and a wooden ferry 12 cm x 5 cm completed the equipment.

Statement of Problem

This ferry takes four cars across each trip. "Show me how you would find out how many trips the ferry makes to get all these cars across the river."

(This is an example of measurement division.)

Problem VIII

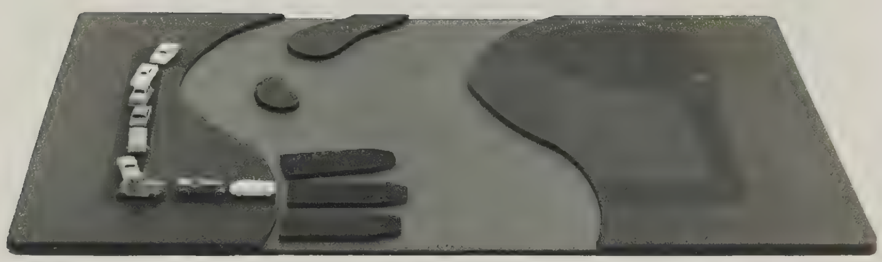

PLATE VIII

Materials

The base board of the previous problem, and three ferries and nine cars.

Statement of Problem

"If these ferries take all the cars across the river in one trip, and all carry the same number of cars, show me how you would find out how many cars each ferry has to carry."

(This is an example of partition division.)

Problem IX

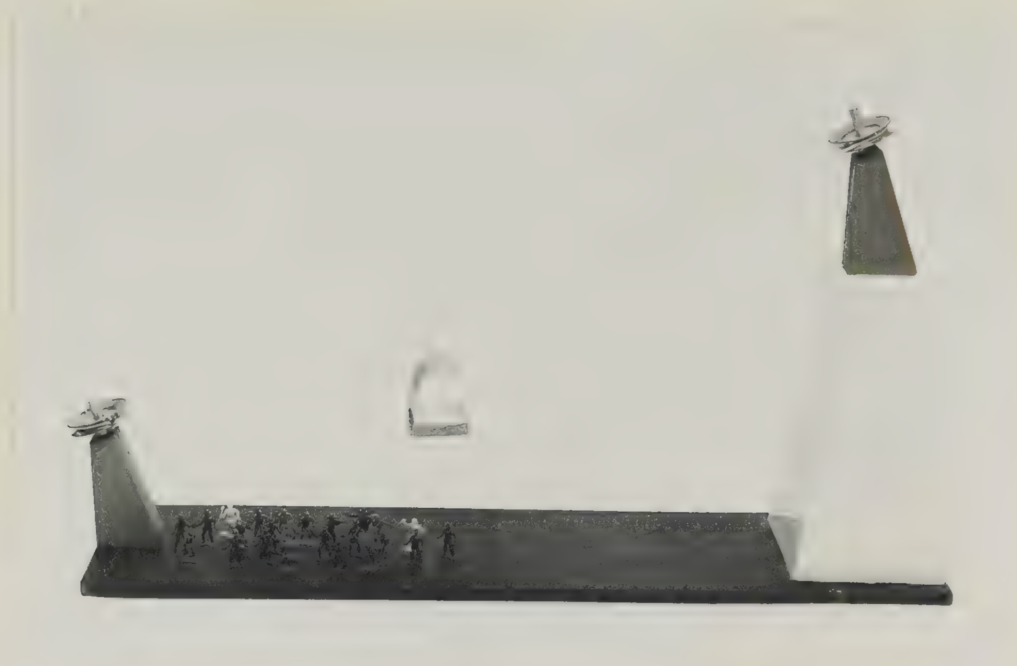

PLATE IX

Materials

A model skytram with one car was waiting to transport 24 people to the top of the mountain.

Statement of Problem

"If the driver takes six people each trip, show me how you would find out how many trips he has to make to get everybody to the top of the mountain."

(This is an example of measurement division.)

Problem X

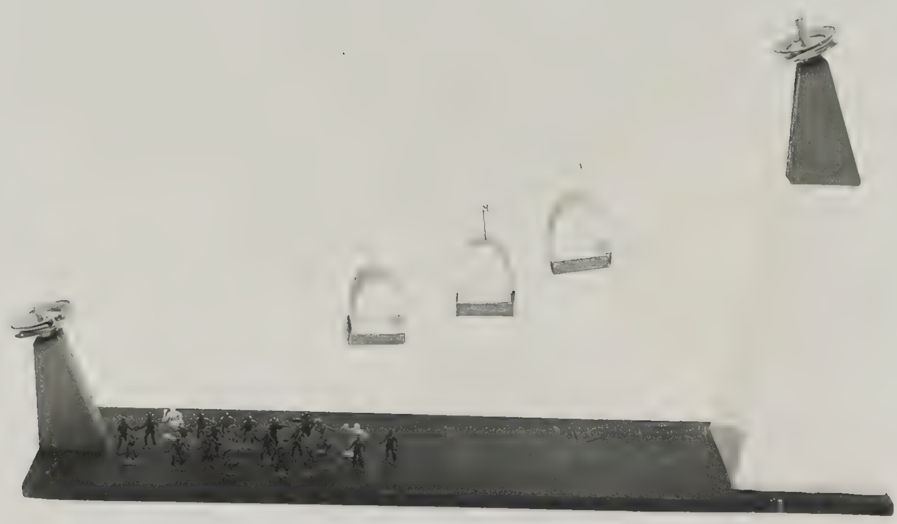

PLATE X

Materials

The model skytram as in the previous problem with three cars fitted, and 24 people.

Statement of Problem

"If the skytram carries all these people up in one trip, and the driver puts the same number of people in each cablecar, show me how you would find out how many people he puts in each car."

(This is an example of partition division.)

TABLE 4

SUMMARY OF PROBLEM-SOLVING SITUATIONS

Problem	Measurement Division	Partition Division
Fence Problems	1. How many fences just like that one can you make with that truck load of poles?	2. If a farmer makes three fences all the same with that lot of poles, how many poles will go in each fence?
Animal Problems	3. If four animals are put in each pen, how many pens will be needed for that lot of animals?	4. If a zookeeper puts all these animals into three cages, so that there is the same number in each, how many animals will go in each cage?
Crane Problems	5. If the crane takes up loads of four blocks, how many loads will be needed to take all those blocks up to the roof?	6. If this number of blocks are taken up in three loads, how many will be in each load?
Ferry Problems	7. If the ferry takes four cars in each trip, how many trips will take this lot of cars across the river?	8. If these three ferries take the cars across in one trip, all carrying the same number of cars, how many cars will be on each ferry?
Sky Tram Problems	9. The cable car carries six people on each trip. How many trips will it take to get this group of people up to the mountaintop?	10. This skytram has three cablecars. If these people are taken up in one trip, each car carrying the same number of people, how many people will be in each car?

The mathematical sentences which could be used to describe these problems could be stated thus:

TABLE 5

EQUATIONS CORRESPONDING TO THE PROBLEMS

PROBLEMS	MEASUREMENT	PARTITION
Fence problem	$32 \div 4 = n$	$9 \div n = 3$
Animal problem	$16 \div 4 = n$	$24 \div n = 3$
Crane problem	$20 \div 4 = n$	$15 \div n = 3$
Ferry problem	$12 \div 4 = n$	$9 \div n = 3$
Skytram problem	$24 \div 6 = n$	$24 \div n = 3$

IV. THE PILOT STUDY

In order to test the effectiveness of the materials to generate behaviors, and the procedures to be used in the investigation, a pilot study was conducted with three boys aged five, six and eight years respectively. These boys were children of neighbours and appeared to be relaxed and unembarrassed by both the situation and the investigator.

The pilot study afforded an opportunity to test the effectiveness of:

a. the verbal instructions for each problem,

b. the ease of manipulation of the materials,

c. the recording techniques of the investigator.

Clarification of expression with the problems was effected, and the need to see that each subject (S) understood the terms used (such as "pen" instead of "paddock", load and trip) was reinforced.

The use of a cameraman to assist the investigator appeared necessary, although as the study proceeded, it was possible to manage without this assistance. Video camera and recording techniques were practised to ensure an accurate recording of the behaviors generated by the problems.

Although the pilot study revealed that the children took longer than was thought necessary to arrive at a solution, no modifications were made of the problem situations, nor the number of problems which were to be solved by the subjects.

V. PROCEDURES WITH THE PROBLEMS

The school science room was made available for this investigation. Familiarity with this room helped to make the boys feel comfortable and at ease with the investigator and with the recording equipment, both factors being considered desirable for this study.

A primary school library table was used to support the materials for each problem. A microphone was mounted on the table to record each subject's voice. A Sony Video Camera was positioned to obtain the best field of vision possible. New format half-inch

magnetic tape was used to record the behaviors by means of a Javelin Videotape Recorder. An assistant was used to operate the equipment in order to reduce the amount of extraneous movement by the investigator.

The subject was allowed to play with the materials for each problem, and thus became familiar with the manipulative skills needed to perform the task. This activity preceded each task, and permitted the investigator to introduce a sample of the vocabulary necessary to the task.

The problem was posed by the investigator, who followed, as closely as possible, an identical pattern with each problem and with each subject. The instructions given to each child were: "Listen carefully while I tell you a problem. After I have repeated it, you can then use these materials, if you wish, to find an answer to the problem. Now do not touch the materials until I have finished repeating the problem."

The behaviors exhibited by the subject as he proceeded to try to obtain a solution were then recorded.

The investigation commenced on May 7, 1973, and proceeded for the next three weeks. Each grade one boy was observed during the first session of the day. After completing the grade one study, the grade two and three boys were observed in two sessions each day, the first between 9:00 a.m. and 10:30 a.m., the second between 10:45 a.m. and 12:15 p.m.

VI. THE ANALYSIS

The problem-solving data collected in the school were recorded on New format half-inch videotape and later were analysed to:

a. find a catalogue of behaviors for each of the subjects belonging to each grade,

b. identify differences in problem-solving behaviors between the subjects in each grade,

c. generate classes of problem-solving behaviors.

From the pilot study and a review of the literature, the following list of expected behaviors was drawn up. This included:

counting	verbalizing
measuring	gestures
partitioning	contemplating
calculating	observing

Tables found in the next chapter record occurrences of these and other behaviors for each problem and for each student.

VII. THE VERBAL PROBLEMS

At the conclusion of the third week, the investigator administered a verbal test to all pupils in grades one, two and three in the school. This test (TEST A in APPENDIX A) consisted of ten problems to be solved by any means, mentally, or by writing, or by drawing. The problems were alternately measurement and partition division types.

Before taking the test, the pupils were asked to find a solution to each problem by some means, by writing, by working with sets of objects, or by drawing pictures. The instructions given were:

"Listen carefully to each problem. See if you can find an answer. You may draw pictures or sets on the paper if that will help you solve the problem. Do not write anything until I have repeated the problem. If you cannot work it out, don't worry. Be ready to try the next one."

Because grade one pupils were unfamiliar with this form of testing, a sample problem was solved, with the class participating in the discussion. With grade one classes in particular, it was necessary to repeat the statement of each problem three or four times. A time interval sufficient for each child to complete a problem if he wanted to do so was provided. Children who did not understand the problem were encouraged to ask questions about it before they wrote down their answers.

In constructing the test, the criteria adopted for "good" problems were kept in mind. The investigator attempted to ensure that the problems related to experiences familiar to most children. The problems are recorded in Appendix A.

Of the eighteen boys who experienced the problem solving situations, one first grade and two third grade boys were absent on the day the verbal test was administered. To complete the record, this test was given a week later to those three boys who were absent on the first occasion. This was written in the school science room, the same room as that used for the performance of the

nonverbal problem-solving tasks.

Monty, the grade one boy, had a result which was so much better than the grade one results, that a second test was administered to see if some maturational factor could explain the disparity. This test (TEST B) is included in Appendix A.

The performances of the 18 subjects of this investigation were compared with those of a random sample of 18 boys, six from each of the three grades. The random sample was obtained by using a table of random numbers (Yamane, 1967, p. 908).

A DERS ANOVA 15 program was used to obtain a one-way analysis of variance for the data. The results of the computer analysis may be found in Appendix B.

VIII. VALIDITY AND RELIABILITY

Discussions were held with several mumbers of the Mathematics Education staff of the Faculty of Education, University of Alberta, to ascertain the face validity of the ten problems constructed for this investigation. All consultants agreed that these problems appeared to meet the requirements of the criteria for "good" problems outlined earlier in this report. The pilot study also provided behavioral evidence to support the use of these problems.

A doctoral student in the department of Secondary Education, G. Babcock, studied the videotapes of a sample of three students and analysed their behaviors in the problem-solving situations. Her analysis and that of the investigator were compared in order to

determine his reliability to score behaviors. Both analyses were in complete concordance.

To obtain as accurate a record as possible, each tape was viewed a minimum of three times, and in many instances, segments of tapes were analysed four and five times.

CHAPTER IV

RESULTS OF THE INVESTIGATION

The findings of the investigation are reported in this chapter. The aims of the study were to catalogue the behaviors of boys in first, second and third grades in nonverbal problem-solving situations, to observe behavioral differences between boys in the three grades, and to generate, if possible, classes of problem-solving behaviors. The effect of experience with manipulative materials on performance with verbally-stated problems was also to be examined.

The results are reported separately in three sections, the first of which contains a list of the behaviors exhibited by the subjects in the problem-solving situations. The second section describes their behavior patterns. The third section is a record of the verbal test performances of boys in this Elementary School. This included the 18 boys who worked with the related physical situations and all other boys in grades one, two and three who had not been introduced to the physical situations at all.

I. OBSERVATIONS OF BEHAVIORS

The following behaviors were observed as the subjects interacted with the different problem situations. Tables 6, 7 and 8

TABLE 6

CATALOGUE OF BEHAVIORS AND FREQUENCY OF PROBLEMS

IN WHICH THESE WERE OBSERVED

GRADE ONE BOYS

Behaviors	11	12	13	14	15	16
Partitioning	3	4	2	3	0	1
Counting	10	9	10	10	9	10
Measuring	6	5	5	5	3	4
Miming	4	0	0	1	0	1
Observing	0	1	3	2	2	2
Classifying	2	2	1	2	2	2
Calculating	0	0	2	0	0	0
Verbalizing	0	4	6	9	9	2
Sighing	1	1	2	2	4	0
Gestures	4	2	6	5	6	2
Fantasizing	1	0	3	2	0	0
Comparing	5	0	1	4	2	5
Contemplating	4	6	7	3	4	8

TABLE 7

CATALOGUE OF BEHAVIORS AND FREQUENCY OF PROBLEMS

IN WHICH THESE WERE OBSERVED

GRADE TWO BOYS

Behaviors	21	22	23	24	25	26
Partitioning	2	4	4	2	2	2
Counting	10	10	10	8	10	10
Measuring	2	5	5	5	6	6
Miming	0	0	0	0	2	0
Observing	2	3	3	1	1	2
Classifying	2	0	2	2	2	2
Calculating	0	0	0	1	0	1
Verbalizing	6	1	2	6	10	4
Sighing	1	0	0	2	3	1
Gestures	5	3	1	6	2	0
Fantasizing	2	0	0	0	4	2
Comparing	2	2	2	1	1	0
Contemplating	6	5	4	4	6	2

TABLE 8

CATALOGUE OF BEHAVIORS AND FREQUENCY OF PROBLEMS

IN WHICH THESE WERE OBSERVED

GRADE THREE BOYS

Behaviors	31	32	33	34	35	36
Partitioning	2	4	5	5	4	4
Counting	9	10	8	10	9	10
Measuring	5	5	5	5	5	6
Miming	0	0	0	0	1	0
Observing	0	0	0	0	1	0
Classifying	2	2	1	1	1	1
Calculating	3	2	4	0	5	6
Verbalizing	2	1	3	6	9	4
Sighing	4	2	0	2	0	0
Gestures	2	0	1	4	3	1
Fantasizing	0	0	0	0	5	0
Comparing	1	0	1	1	1	2
Contemplating	6	6	2	3	1	3

show the number of problems in which each boy exhibited the behaviors. A general description of each behavior appears at the end of this section. In these descriptions, a boy's grade will be indicated in parentheses after his name. Thus, Ernie (Gr. 1) refers to Ernie who is in grade one. For convenience a code for the name of each boy has been devised. The first numeral of the code signifies the grade and the second numeral indicates the videotaping order for that boy. For example, '14' indicates that Timothy was in grade one and was fourth in the filming order. Similarly, tables of frequencies of problems in which these behaviors occurred have been constructed for grade two and grade three boys.

The behaviors listed above were generally manifested in the following actions.

a. Partitioning occurred when a boy formed a number of sets and allotted the same number of items to each. This occurred in a variety of ways, in some cases one item at a time, and in other cases, several items at a time. No subject used a completely systematic way of partitioning.

b. Counting took place when the number of items in a set was determined. This was usually manifested by counting aloud, or by pointing with a finger, or by counting with a hand gesture. Thus, counting could occur with or without gestures.

c. Measuring or grouping in equal sized groups was manifested by the formation of sets of a particular size. It was generally accompanied by counting.

d. <u>Miming</u> occurred when a boy deliberately acted through a situation with or without the actual manipulation of materials. For example, when cars were moved across the river and attention to such details as keeping the cars on the road, and when, by careful maneuvering the cars were moved into the car park, this action was regarded as miming.

e. <u>Observing</u> was manifested by a boy's withdrawal from the material after performing an action, and a review of that action on the problem situation. There were usually head movements from one side to the other as the situation was summed up.

f. <u>Classifying</u> took the form of making sets according to some physical criterion, such as a particular kind of animal. For example, if a set of animals consisted of all horses, while another consisted of pigs, the arrangement is likely to have been based on the species of animal present, and its formation dependent on classification.

g. <u>Calculating</u> occurred when an answer was obtained without the use of the manipulative materials. It could be observed through overt behaviors such as whispering the calculations, a counting with the fingers, and a staring into space accompanied by a combination of these.

h. <u>Verbalizing</u> took a variety of forms such as talking to oneself about the problem and asking questions of the

investigator, or talking to the objects.

i. Sighing was an obvious overt action.

j. Gestures were also obvious overt behaviors, and often took the form of hand or finger actions accompanying counting. Head movements were similarly identified. Hand movements in which materials such as the wooden blocks were placed on the crane were not identified as gestures, while hand movements which substituted for the movement of cars were identified as miming. Actions such as stroking one's face with a hand, or shrugging the shoulders were classified as gestures.

k. Fantasizing occurred when a boy went to extremes in miming or referred to his pet puppet and what he would say to it, and conjectured on what his friend would think of what he was doing at that moment.

l. Comparing physical situations was shown by stooping down and sighting the tops of fences to see if they were the same height, and by placing two piles of blocks side by side while comparing their heights with the hand placed across them.

m. Contemplating was accompanied by a variety of overt actions, such as looking steadfastly at the material with no other action, or looking away from the model and staring thoughtfully into space.

II. RESULTS OF OBSERVATIONS FOR EACH PROBLEM

Problem I

Each boy was asked to find how many fences with four poles he could make with 32 poles. The task required each boy to fit posts into the holes in the wooden base, and to slide poles into these posts (See plate I page 40).

Two grade one boys, Ernie and Danny, proceeded to make fences with eight poles in each. Danny removed the surplus poles when the problem was recalled, and then completed forming fences with four poles in each section. Ernie, on the other hand, did not appear to see that his fences with eight poles were not the same as the speciman fence with four poles in it. Although he could repeat the statement of the problem, he was content to retain his arrangement of poles.

Monty (Gr. 1) completed seven fences correctly, and then made three others with two poles in one fence and one pole in the others. On recalling the original problem he correctly completed the eight fences.

Two grade three boys had initial difficulties with the number of poles used for each fence. Peter completed all fences correctly except the first, which had five poles, and the last, which had three poles. This was likely an error in counting which he noticed when he was asked to check the number of poles in each

fence. Paul, on the other hand, made two fences with eight poles in each, and, on his own initiative, decided that these were incorrect. He then became absorbed with the idea of completing fences in every possible space on the board, and this led to his construction of fences with three poles in each. The recall of the original problem enabled him to correct the assembly of poles.

The number of poles picked up at any time was another behavior variable. Andy (Gr. 1) and Scott S. (Gr. 2) consistently picked up four poles each time from the pack. Generally one pole at a time was selected by the other boys.

The grouping of poles into sets of four was identified as a measurement operation and as such was exhibited initially by all but those boys mentioned above. The investigator had expected that some boys would have grouped the poles into sets of four without actually assembling the fences. However this did not occur.

Gestures which occurred during this problem were associated with other behaviors such as

a. counting the poles in a fence, for example pointing with the finger,
b. counting the number of fences, for example, nodding the head, lip movements accompanied by eye movements, and audible and sub-audible recitations of counting numbers.

Verbalizing varied considerably from boy to boy within any one grade and also varied from grade one to grade three. The personality of a boy also had some bearing on the amount of verbalization which took place.

For example, James M. (Gr. 2) asked for some clarification of the problem. "Do the fences have to have four poles like that one?". Again, Monty (Gr. 1) said as he was putting a fifth pole into a fence, "I don't want to do that."

Ronald (Gr. 3) fantasized thus: "I don't know what I'm doing here! . . . Sometimes I think I'm losing my mind. There! I've finished." He also referred to conversations he had with a puppet he called "Scooby-doo."

Table 9 shows the behaviors generated by this problem with each of the boys. Clearly, counting, measurement, comparing and contemplating are common to all grades, while gestures appear to be more general in grade one and two boys, miming and classifying did not occur in this problem. Verbalizing did not appear to occur in grade two boys as much as in grade one and grade three boys.

Problem II

> Three identical fences are to be constructed from nine poles. How many poles will be in each fence? This problem required the partitioning of nine objects into three equal sets. Nine poles were laid on the base. Nine posts were available for use.

Three grade one boys, Danny, Timothy and Troy commenced making fences with four poles in each. On being reminded of the problem to be solved, Danny and Timothy adjusted the fences to

TABLE 9

BEHAVIORS SHOWN BY BOYS IN PROBLEM I

Behavior	Grade 1						Grade 2						Grade 3					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Partitioning																		
Counting	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x
Measurement	x		x	x	x		x	x	x	x	x	x	x	x	x	x	x	x
Miming																		
Observing				x	x	x	x		x									
Classifying																		
Calculating				x										x			x	x
Verbalizing		x		x	x	x				x	x			x		x	x	x
Sighing	x				x								x			x		
Gestures	x	x	x	x	x		x	x	x	x	x						x	
Fantasizing							x											
Comparing	x	x		x	x	x	x	x	x	x	x		x		x	x	x	x
Contemplating	x	x	x	x		x				x	x	x	x	x	x			

contain three poles in each. Troy, however, lost interest after assembling two poles in each of three fences, leaving himself holding three poles. He appeared at a loss as to what to do with them. Ernie (Gr. 1) assembled eight poles in one fence and one pole in another.

All boys except Ernie (Gr. 1) and Ronald (Gr. 3) showed evidence of counting, either by fingering, nodding, by lip or eye movement, or by whispering. Ronald may have counted without any overt sign. In each case they arrived at a very quick solution.

Calculating was often difficult to observe, although it was evident from the confident manner in which the fences were constructed. Scott S. (Gr. 2) immediately assembled the three fences each with three poles. Peter and James C. (Gr. 3) offered 'three' as an answer without any manipulation of the poles at all. They proceeded to confirm their calculation by completing the construction. Only grade three boys calculated.

Of the 18 subjects, one grade one, four grade two, and six grade three boys showed some evidence of a partitioning behavior by constructing fences with three poles in each but this behavior was not consistent. However, only Paul (Gr. 3) did his partitioning by making groups of three poles on the table. He did not assemble the fences.

Three grade one, three grade two, and two grade three boys talked as they attempted to solve the problem. Monty, Ernie and Troy talked aloud about the problem. Monty said, "I think I've got a problem here -- should be two poles but I've got two more."

(At the time he was confusing posts with poles). He proceeded to complete the problem successfully.

An unexpected behavior appeared with this problem, and this seemed to be an external manifestation of frustration and annoyance with oneself. Lyle had placed all nine posts in the holes. He constructed three fences by putting a pole between two posts, thus forming long fences with two poles end to end. This was quite different from the concept of 'fence' established in problem I. His difficulty was, what to do with the remaining three poles he held? If he inserted one in each fence, then the two sections of a fence were different. He proceeded to reassemble the poles in a variety of positions none of which satisfied him. This was accompanied by loud sighs and shrugging of shoulders. When he was reminded by the investigator of the way in which fences were contracted previously, Lyle approached this task with less sighing and completed it to his satisfaction.

Although contemplation seemed characteristic of the grade one and two boys, only James C. (Gr. 3) appeared to spend any time thinking about the task. This thinking was characterised by a lack of accompanying action and a partial closing of the eyes in what appeared to be a determined effort to think through the problem.

There was no evidence of miming, classifying or fantasizing with this problem. Table 10 lists the observed behaviors for this problem.

TABLE 10

BEHAVIORS SHOWN BY BOYS IN PROBLEM II

Behavior	Grade 1						Grade 2						Grade 3					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Partitioning		x						x	x	x		x	x	x	x	x	x	x
Counting	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x		x
Measurement	x			x		x												
Miming																		
Observing				x						x							x	
Classifying																		
Calculating													x	x	x			x
Verbalizing			x	x	x		x	x		x	x						x	x
Sighing	x		x	x			x									x		
Gestures	x		x	x	x			x		x								
Fantasizing																		
Comparing	x					x												
Contemplating	x	x	x		x	x	x	x	x		x		x					

Problem III

Sixteen farmyard animals are to be put into pens with four animals in each pen. How many pens are needed? The materials available allowed each boy to make pens and assemble the animals by fours.

The most favored behavior appeared to be classification. There were four animals in each class, namely four horses, four pigs, four ducks and four sheep. To determine the number of pens needed, it was simply a matter of finding how many classes of animals were present. Carl (Gr. 2) asked if the animals had to be the same in each pen, and like several others, suggested "four" and proceeded to test this answer by making the pens and inserting the animals in the pens. Only Peter (Gr. 3) arrived at his solution by measuring off sets of four animals selected at random, without assembling the walls of the pens.

Monty, Timothy and Ernie of grade one, Chris and James M of grade two, and Ronald (Gr. 3) were constant talkers, chatting to the investigator and to themselves about the situation and about the animals as they proceeded with the task.

Gestures were associated with counting and generally took the form of pointing to each animal in its pen to make sure that the requirements of the problem were met. In the case of Peter (Gr. 3) the gestures were simply placing a spread-out hand over each of the sets of animals to determine the number of pens required.

Table 11 shows that counting, measuring and classifying were dominant behaviors, while miming, observing and calculating were absent.

TABLE 11

BEHAVIORS SHOWN BY BOYS IN PROBLEM III

Behavior	Grade 1						Grade 2						Grade 3					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Partitioning				x														
Counting	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Measuring		x	x					x	x	x	x	x		x	x	x	x	x
Miming																		
Observing																		
Classifying	x	x	x	x	x	x	x		x	x	x	x	x	x	x	x		x
Calculating																		x
Verbalizing			x	x	x					x	x						x	
Sighing			x								x			x				
Gestures	x		x	x	x			x									x	
Fantasizing	x		x														x	
Comparing						x												
Contemplating		x					x		x				x					x

generated by the problem.

Problem IV

> Twenty four animals, in sets of three of a kind, are to be housed in three cages. How many animals will be put in each cage? The solution to this problem required each boy to make a departure from a simple classification by genus, and forced him to classify by number.

With the exception of Monty (Gr. 1), Carl (Gr. 2), Scott G. (Gr. 2), Paul (Gr. 3), and Peter (Gr. 3), the boys proceeded to classify the animals into sets of three: three apes, three bears, three buffalo, etc. This provided them with the basis for an initial partitioning into three equivalent sets. Some attempt was made to put animals into the same cage according to compatibility. Eventually all but one boy, Mark (Gr. 3), reached the stage where the animals were put into the cages one at a time, in turn, until the animals remaining from the initial distribution were all accommodated. Mark, a boy of very few words, inserted three animals in each cage, and then refused to proceed. He stated, on being questioned, that more cages would have to be made, since it was not possible to instal two different kinds of animals in the one cage.

Peter (Gr. 3) and Paul (Gr. 3) were two exceptions to the classification behavior listed above. They formed equivalent sets by partitioning into groups of six regardless of species, and then

added one animal to each group until all had been distributed. In both cases the grouping appeared to be random. Furthermore, they did not assemble the walls of the cages. Prior to his partitioning, each boy counted the total number of animals present, thus arriving at a convenient size for the first subsets formed.

Many inaccuracies occurred in the counting, by failing to make correct one-to-one correspondences when adding animals to cages, and reciting the counting numbers. This was not restricted to boys in grade one and two, for Ronald, Lyle, and James C., all in grade 3, were also inaccurate when counting. As well as the type of error mentioned, another mistake was the failure to name the correct number when a count was resumed.

Fantasizing took the form of talking to the animals as they were placed in their pens. James M. expressed concern that the rhinoceros would eat the apes.

Table 12 shows that counting, classifying, verbalizing gestures and contemplating were prevalent. There was no miming, and very few observing, sighing and comparing behaviors.

Problem V

> Twenty blocks are to be lifted by a crane to a building roof in groups of four. The task is to find how many groups of four blocks are lifted by the crane. This required the measuring of sets of four.

Andy, Ernie and Monty, from grade one confidently proceeded

TABLE 12

BEHAVIORS SHOWN BY BOYS IN PROBLEM IV

Behavior	Grade 1						Grade 2						Grade 3					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Partitioning	x						x	x							x	x		x
Counting	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Measuring											x							
Miming																		
Observing					x	x												
Classifying	x	x		x	x	x			x	x	x	x	x	x	x		x	
Calculating													x				x	x
Verbalizing		x		x	x					x	x		x		x		x	x
Sighing											x		x					
Gestures	x	x	x	x		x	x			x	x		x			x		
Fantasizing			x	x							x					x		
Comparing	x					x												
Contemplating		x	x	x	x	x	x	x		x	x			x	x			

to make sets of four blocks on the table and then counted the number of sets formed. Peter and Paul, of grade three, also followed this procedure. With the exception of the last two named, all boys took the blocks up on the crane.

Counting the number of groups of four was a haphazard affair, resulting from the unsystematic arrangement of blocks such as that made by Danny, Timothy and Troy (Gr. 1). The remaining boys placed the blocks in neat piles which facilitated counting. Chris (Gr. 2), simply counted the total number of blocks (20), and gave a correct answer in a few seconds. When asked how he arrived at his answer, he said, "I just counted them." He then proceeded to sort the blocks into groups of four to check his answers.

Gestures other than those associated with counting were evident with Monty (Gr. 1), Chris (Gr. 2), James C. and Ronald (Gr. 3), who demonstrated an orderly approach to the placement of blocks in piles. These were patted into blocks or towers, which facilitated the counting of the groups.

Verbalizing was again prominent, and consisted of talk about the task, speculation about the reason for taking the blocks up to the roof, while fantasizing consisted of the crane driver's comments about the loading of the crane, and where he was to put down the load.

Table 13 shows those behaviors exhibited by the boys when working on this problem. Counting, measuring and verbalizing prevailed, and were consistent over the grades. No partitioning or classifying was observed, and there were only a few instances of miming, observing, sighing, fantasizing and comparing. The latter behaviors

TABLE 13

BEHAVIORS SHOWN BY BOYS IN PROBLEM V

Behavior	Grade 1						Grade 2						Grade 3					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Partitioning																		
Counting	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Measuring	x	x	x		x	x		x	x	x	x	x	x	x	x	x	x	x
Miming	x										x							
Observing								x	x			x						
Classifying																		
Calculating												x		x				x
Verbalizing		x		x	x		x		x		x					x	x	
Sighing					x							x						
Gestures			x							x			x				x	
Fantasizing				x														
Comparing	x								x									
Contemplating			x			x	x	x					x	x				

were shown by boys in grades one and two.

Problem VI

> This problem required a set of 15 blocks to be partitioned into three equal sets, so that the crane could take the blocks to the roof in three equal loads.

This problem carries a great deal of confusion because the boys may not have been able to visualize the three loads, since the same crane was used again and again. Of the eighteen boys, eight took the blocks up three at a time, which suggested the existence of a mental set on the number 3 (the number of trips to be made). Andy (Gr. 1), Scott G. (Gr. 2) and David (Gr. 2) wanted to take up groups of four blocks which indicated a reversion to the previous problem. However, Andy then went on to rearrange the blocks on the table into three groups of five. Ernie (Gr. 1) tried to arrange the blocks into three piles, but did not seem able to depart from a 4, 5, 6 combination.

David (Gr. 2) put four blocks on the crane, thought for several seconds, and then added another block. Without proceeding to raise the crane, he declared that there would be five blocks on each trip. He then counted the sets of five, to confirm that there would be three groups.

The tally of boys who exhibited specific behaviors in solving problem VI is shown in Table 14. Partitioning behavior

TABLE 14

BEHAVIORS SHOWN BY BOYS IN PROBLEM VI

Behavior	Grade 1						Grade 2						Grade 3					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Partitioning		x							x					x	x	x	x	
Counting	x	x	x	x	x	x	x	x	x		x	x		x	x	x	x	x
Measuring	x					x							x					
Miming											x						x	
Observing							x											
Classifying																		
Calculating															x		x	
Verbalizing			x	x	x	x	x			x	x	x			x	x	x	
Sighing										x			x	x				
Gestures			x		x		x			x						x		
Fantasizing											x	x						
Comparing				x	x	x	x	x										x
Contemplating	x	x	x			x	x	x	x	x	x	x	x	x		x		x

demanded by this problem was mainly demonstrated by grade three boys, only one from each of the other grades showing this action. However measuring, which was not required here was used by three boys.

Contemplating was very evident throughout the grades, and no attempt was made to classify. Instances of miming, observing, calculating, sighing and fantasizing were few in number, and were exhibited only by boys in grades 2 and 3.

Problem VII

> In the first of the ferry problems each boy is asked to find how many trips a ferry has to make to transport 12 cars across the river, if it carries four cars on each trip. The cars are lined up waiting for the ferry to be loaded.

Counting, accompanied by a variety of gestures, was displayed by all boys. Finger movements, hand movements, and eye movements were all observed and were not specific to any particular grade. James C. simply counted the cars and declared "three, er, no -- four."

Measuring was observed at two levels. At the first level cars were loaded onto the ferry in sets of four, and transported across to the other side of the river. Twelve boys including James C., Lyle and Ronald from grade 3, exhibited this particular behavior.

At the second level, measuring occurred without the cars being moved. Five boys including Ernie and Andy of Grade 1 found the number of sets of four either by counting with the finer or spanning with the hand as they counted "one, two, three."

Miming was not a prominant behavior in this problem situation. However, Danny, Timothy and Troy, grade 1 boys, went to extreme lengths in manipulating the cars and ferry. Having moved the cars off the first ferry, it was necessary to move these cars further along the road to allow the second ferry load to disembark. It took several attempts before the cars were all moved into the parking lot, meticulous care being taken each time to keep the cars in line. At the other extreme, Ronald took great delight in manipulating the cars and ferries, at the same time emitting sound effects such as screeching brakes and horn blasts to simulate an actual situation. This was quite consistent with Ronald's tendency to fantasize. The behaviors exhibited by the boys in this problem situation are listed in Table 15. No partitioning, classifying, calculating, or comparing behaviors were exhibited. Miming was displayed by grade 1 boys only. Observing, sighing and fantasizing were infrequently displayed during the solving of this problem.

Problem VIII

Nine cars were waiting to cross the river on three

TABLE 15

BEHAVIORS SHOWN BY BOYS IN PROBLEM VII

Behavior	Grade 1						Grade 2						Grade 3					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Partitioning																		
Counting	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Measuring	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x
Miming	x			x		x												
Observing		x						x										
Classifying																		
Calculating																		
Verbalizing										x	x	x	x		x	x	x	
Sighing		x			x					x								
Gestures			x		x					x					x	x		x
Fantasizing																	x	
Comparing																		
Contemplating		x	x			x								x		x		

ferries. The problem was to find out how many cars went across on each ferry; all ferries were to carry the same number of cars.

Troy, Timothy and Scott S. commenced by loading four cars on each of two ferries leaving the third ferry to carry one car. This probably indicated a mental set on the number of cars used in the previous problem. Although the cars were obviously unevenly loaded, this did not seem to disturb Timothy (Gr. 1), who indicated that the ferry could carry more cars. Troy (Gr. 1) and Scott S. (Gr. 2) did adjust the number of cars on each ferry.

In one case, only that of Paul (Gr. 3), did the partitioning take the form of placing one car in turn on each of the ferries, and repeating the procedure until the cars were all accounted for. Nine boys, namely Danny, Andy and Monty from grade one, Scott G. and David from grade two, and James C., Mark, Ronald and Lyle of grade three, partitioned the cars into three sets of three. The remainder offered the answer before making any move to verify their solution. The investigator questioned them to determine how they arrived at their answer. From their replies it appeared that the partition had been made mentally by observing the number of ferries and the number of cars.

Table 16 shows the pattern of behaviors exhibited by the boys in this problem. Partitioning was displayed by three boys in each of the grades 1 and 2, more than in any of the previous problems where this behavior was expected. Counting dominated the action, while classifying and sighing were completely absent. The remaining behaviors were exhibited only infrequently. Calculating, when it

TABLE 16

BEHAVIORS SHOWN BY BOYS IN PROBLEM VIII

Behavior	Grade 1						Grade 2						Grade 3					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Partitioning	x	x	x					x	x		x		x	x	x	x	x	x
Counting	x		x	x	x	x	x	x	x		x	x	x	x		x	x	x
Measuring				x								x						
Miming	x																	
Observing					x						x	x						
Classifying																		
Calculating										x					x		x	
Verbalizing		x		x	x		x				x							
Sighing																		
Gestures				x		x	x									x		
Fantasizing											x						x	
Comparing				x														
Contemplating				x	x	x				x								

occurred, was used by the older boys.

Problem IX

This problem required the boys to form the cablecar passengers into sets of six to find the number of trips needed to transport them to the mountain top.

As with the previous problems, counting was a behavior exhibited by all boys. Similarly, measurement was a common behavior except for Chris (Gr. 1), who counted the total number of people and declared the answer to be '4' after a brief mental calculation. Danny (Gr. 1) found it difficult to count groups of six into the cable car, and placed people in it rather haphazardly. Later he reviewed the situation, and formed groups of six on the table. James M. (Gr. 2) found it difficult to recall how many trips the cablecar made. He verbalized this: "I counted my first one, then my second. My brain is thinking that it could be 5." Scott S. (Gr. 3) also spoke about the equipment. When a model fell out of the car, he commented, "The man jumped out as we turned the corner." Other questions asked by the boys concerned the placement of the model when they reached the top of the mountain. Thinking or contemplation was again characterized by a cessation of activity for several seconds, which was then followed by a burst of activity.

Table 17 shows that counting, measuring, verbalizing and contemplating were the major behaviors exhibited. Partitioning and comparing were completely absent. Classifying, exhibited only by one boy, was due to the plastic figures used for this problem, and was not attributable to the structure of the problem itself. The

TABLE 17

BEHAVIORS SHOWN BY BOYS IN PROBLEM IX

Behavior	Grade 1						Grade 2						Grade 3					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Partitioning																		
Counting	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Measuring		x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x
Miming	x																	
Observing			x					x	x									
Classifying							x											
Calculating			x														x	x
Verbalizing		x	x	x	x		x		x		x	x				x	x	
Sighing					x													
Gestures							x											
Fantasizing											x						x	
Comparing																		
Contemplating	x	x	x	x				x	x		x					x		x

other behaviors occurred infrequently.

Problem X

In this problem, 24 people were to be transported in three cable cars, each holding the same number of people.

Whereas all boys but one showed some evidence of making three equivalent sets, Ernie (Gr. 1) spent several minutes forming sets of different sizes, finally arriving at a four sets of six formation. When his attention was drawn to the statement that only three sets were needed, the fourth set was then divided equally between the other three sets. The method of partitioning varied considerably, from Peter (Gr. 3) who formed six sets of four on the board and rearranged them to form three sets of eight, to Carl and James M. (Gr. 2), Mark (Gr. 3), Timothy, Chris and Troy (Gr. 1), all of whom formed three sets of six and then added two to each set. David (Gr. 2) chose the seven plus one format for his solution. Again, Lyle (Gr. 3) lost count and finally formed the three equal sets by adding three sets of two to three sets of six. Ernie and Monty (of grade one) both formed groups which were nearly equal, and by counting and recounting, were able to adjust the groups so that they had equal numbers in each.

Verbalization, for instance, took the form of talking about the equipment, while Ronald discussed his visits to Banff and Jasper, and also talked to the passengers of the sky tram.

The behaviors exhibited by the boys in solving problem X are

displayed in Table 18. Partitioning, counting and verbalizing were the main behaviors observed. In particular, partitioning in some form was displayed by seventeen of the eighteen boys. Miming, observing and classifying were not displayed at all, and the remaining behaviors appeared only briefly.

Other Results

The mean time and the range of times taken by each grade to arrive at a solution for each problem is shown in Table 19. It appears that, with few exceptions, the older boys solved the problems more quickly than the younger boys. Exceptions occurred in problem II, where the mean time for grade 3 exceeded that for grade 2, and in problem VI, where the mean time for grade 2 exceeded that for grade 1.

The frequencies listed in Table 20 indicate for each grade the number of problems in which the particular behavior actually occurred. There were six boys from each grade, and each boy solved ten problems. Thus the absolute maximum score for any behavior would have been 60.

The most frequently displayed behavior counting, was also consistently applied by all grades.

The capacity of a problem to generate behaviors was determined by the total child-behavior scores obtained from tables 9 throught 18. These are recorded in Table 21.

TABLE 18

BEHAVIORS SHOWN BY BOYS IN PROBLEM X

Behavior	Grade 1						Grade 2						Grade 3					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Partitioning	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x
Counting	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Measuring																		x
Miming																		
Observing																		
Classifying																		
Calculating													x					
Verbalizing		x	x	x	x		x				x	x				x	x	x
Sighing					x						x		x					
Gestures	x				x													
Fantasizing			x				x					x						
Comparing	x			x														
Contemplating	x					x	x				x		x	x				

TABLE 19

TIME SPENT ON PROBLEMS: MEAN AND RANGE FOR EACH GRADE (IN MINUTES)

Problem	Grade 1		Grade 2		Grade 3	
	Mean	Range	Mean	Range	Mean	Range
I	9.0	7 -12	7.0	3 - 9.0	7.0	6 - 9.5
II	6.0	3.5-10	2.5	1 - 5.5	3.0	.5-12.5
III	6.5	4 -10	3.5	.5- 5.0	2.0	.5- 6.0
IV	7.5	4 -13.5	6.0	1.5-10	5.0	1.0- 8.0
V	7.0	2 -15	5.5	1 -11	3.0	<.5- 6.0
VI	6.5	2.5-14.5	8.0	4 -10	5.0	.5- 3.5
VII	3.5	2.5- 4	2.0	.5- 3.5	1.0	<.5- 3.0
VIII	3.0	2.5- 5	1.0	< .5- 2	1.0	<.5- 2.5
IX	4.5	2.5- 6.5	4.5	< .5- 7.0	2.5	.5- 4.5
X	3.5	0.5- 6	2.0	.5- 3.5	1.5	.5- 3.5

TABLE 20

FREQUENCIES OF PROBLEMS GENERATING BEHAVIORS IN BOYS

Behavior	Grade 1	Grade 2	Grade 3
Partitioning	13	16	25
Counting	58	58	56
Measuring	28	29	31
Miming	6	2	1
Observing	10	12	1
Classifying	11	10	8
Calculating	2	2	20
Verbalizing	30	29	25
Sighing	10	7	8
Gestures	25	17	11
Fantasizing	6	8	5
Comparing	17	8	6
Contemplating	32	27	21

TABLE 21

CHILD-BEHAVIOR SCORES FOR THE PROBLEM SITUATIONS

Problem	Measurement	Partition
Fence I & II	94	69
Farm III & IV	71	82
Crane V & VI	65	73
Ferry VII & VIII	62	52
Skytram IX & X	65	61

As a generator of behaviors Problem I can be seen to have the greatest capacity, and Problem VIII the least. Measurement division problems generated more behaviors than the partition division problems. It is clear from Tables 9 through 18 that measurement division problems were successful in generating measurement behavior, and partition division problems were likewise successful in generating partitioning behavior. On the other hand, the measurement problems produced only one partitioning behavior, whereas the partition problems evoked a response of 10 measurement behaviors.

More talking and contemplating occurred during the partition problems, whereas the measurement problems resulted in a greater incidence of comparing and observing behaviors.

Other behaviors which were displayed to approximately the same extent by all grades were measuring, verbalizing, sighing and fantasizing. The frequency of partitioning behaviors increased with grade level, being greatest for third grade. Calculating was almost exclusively exhibited by grade 3 boys. For all other behaviors, the frequency trend was the reverse, namely, greatest for grade 1 and least for grade 3.

The frequencies for the behaviors in Table 20 are not absolute, and give no indication of the total number of time a

particular behavior occurred.

II. THE VERBAL PROBLEMS

The purpose of this part of the study was to compare the performances in solving verbal problems of those boys who had had experience with the nonverbal problem-solving situations (the EXPERIENCED) with those who had not (the INEXPERIENCED). To do this, a test of ten items, five measurement division problems and five partition division problems, was presented orally by the investigator. (See Test A, Appendix A).

At the outset, an important difficulty appeared, namely, the fact that grade one children had had no previous experience with this form of testing. To overcome this, the investigator discussed a sample problem with these children, who suggested various ways in which the problem could be solved. This was followed by a discussion in which the solution was obtained by those various methods which were applicable.

In answering the test questions, some pupils actually drew sets of marks on their answer form and marked off subsets of marks appropriate to the question. Others used some convenient representation such as counting on their fingers, and in one case a set of 25 crayons was used to partition a set of 25 into five equivalent sets. The number of correct responses obtained by each of the 18 subjects is shown in Table 22. As already reported in chapter III, a second verbal test, Test B, was administered orallv two weeks later. The results of this test are also recorded in Table 22, to conpare the performances of the 18 subiects in the two tests.

TABLE 22

TEST RESULTS

NUMBER OF CORRECT RESPONSES FOR SUBJECTS

IN THE INVESTIGATION

Boy	Test A	Test B	Boy	Test A	Test B	Boy	Test A	Test B
11	0	3	21	8	10	31	5	8
12	3	2	22	6	10	32	10	8
13	9	2	23	6	7	33	5	8
14	0	0	24	3	1	34	5	7
15	4	4	25	4	7	35	10	10
16	1	1	26	8	10	36	9	8
$\overline{X}$	2.83	2.00		5.83	7.5		7.33	8.17
S.D.	3.43	1.41		2.04	3.51		2.58	.98

Of the three boys who, because of absence at the time of the first test, were given this verbal test as a small group, Monty (Gr. 1) gained an exceptionally high score. However, in the second test his score was very close to the mean for his group. Possible reasons for such an unusual result include

i. the fact that the test was done under more favorable conditions, namely, in the room where he had solved his nonverbal problems,

ii. the first test situation was, for him, almost a person-to-person test, whereas the second test was presented in the normal class-teacher situation,

iii. no prior discussion of ways to solve problems was held with him, and no sample problem was solved,

iv. the presence of the other older boys may have had a stimulating effect.

To form a basis for comparison with the other boys in grades one, two, and three, who did these tests, a random sample was drawn from this population, containing six boys from each grade. The results obtained by these boys are shown in Table 23. The means and standard deviations for each of these six groups are also listed with the test results.

Another general observation was the tendency for most first and second grade scores to improve in the second test. This trend was not so obvious with the third grade scores. This may have been due to the fact that the test situation was unfamiliar to the classes.

TABLE 23

TEST RESULTS

NUMBER OF CORRECT RESPONSES FOR A RANDOM

SAMPLE FROM EACH GRADE

Boy	Test A	Test B	Boy	Test A	Test B	Boy	Test A	Test B
1A	1	2	2A	5	5	3A	10	10
1B	1	2	2B	4	9	3B	10	10
1C	3	0	2C	3	6	3C	10	9
1D	1	4	2D	6	6	3D	1	4
1E	3	2	2E	8	5	3E	7	8
1F	3	5	2F	6	9	3F	4	3
$\overline{X}$	2.00	2.50		5.67	6.50		7.00	7.33
S.D.	1.09	1.76		2.06	1.97		3.79	3.08

TABLE 24

COMPARISON OF TEST MEANS

Grade	Study Sample		Random Sample	
	Test A	Test B	Test A	Test B
1	2.83	2.00	2.00	2.50
2	5.83	7.50	5.67	6.50
3	7.33	8.17	7.00	7.33

From the one-way analysis of variance recorded in Appendix C, there is no significant difference between the two grade one groups, between the two grade two groups and between the two grade three groups. However, if we consider the three groups of boys in this study, the difference between the means for Test A for the grade one and grade three boys is significant ($p < .05$), but not significant between grade one and grade two, nor between grade three and grade two.

For Test B, the differences between means are highly significant ($p < .0005$) for grade one and grade two, and for grade one and grade three.

Summary of Findings

This chapter contained a catalogue of the behaviors exhibited by eighteen boys in grades one, two and three when placed in nonverbal problem-solving situations. These results were analysed in order that a pattern of behavior responses for each of the ten problems could be established.

The frequencies with which problems generated specific behaviors were tabulated; however, the frequency of any behavior for a particular problem was not noted. For the grade one boys, the most widely exhibited behaviors were counting, measuring, verbalizing, gestures and contemplating. For the grade two boys the pattern of responses was identical with that of grade one. Although these behaviors were also exhibited by the grade three boys, it was particularly noticeable that the number of gestures and the amount of verbalizing and contemplating diminished with the increase in grade level. As a contrast, there was more evidence of partitioning and calculating behavior with grade three boys than with grade one or two boys. In the animal problems, where classifying was a feasible behavior, it was exhibited by almost all the boys, whether or not it had any bearing on the solution to the problem.

The observations recorded show conclusively that these problem situations were successful in generating a wide variety of behaviors in boys from grades one, two and three. Furthermore, specific behaviors appeared to be common to all three grades, namely counting, measuring, verbalizing and contemplating. Other behaviors such as calculating were characteristic of grade three. Again, particular problem settings

seemed to elicit other behaviors such as classifying.

Another feature brought out by the observations is that measurement division problems did, in fact, produce measurement behaviors, while partitioning problems did not always produce partitioning behaviors. In fact, the eliciting of partition behaviors by partition problems was restricted almost exclusively to the grade three boys.

The behaviors generated by these problems could be classified into three broad categories, namely cognitive, affective and motor, with a preponderance of behaviors in the cognitive domain. This was possibly due to the mathematical nature of the problems.

The time spent by the boys in each of the problem-solving situations varied considerably. Nevertheless, two tendencies appeared from the analysis. Firstly, the grade mean time for each problem tended to decrease with the increase in grade level. Secondly, within each grade, the mean time spent in obtaining a solution decreased as the boys proceeded from problem I to problem X.

The results of the verbal problem-solving tests indicate that boys who had experience with the problem-solving situations did not perform significantly better than boys in the same grade who had no such experience.

CHAPTER V

SUMMARY, CONCLUSIONS, IMPLICATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

I. SUMMARY OF THE INVESTIGATION

The purposes of this investigation were:

1. to catalogue behaviors of boys in grades one, two and three, in nonverbal problem-solving situations,
2. to observe differences in problem-solving behaviors of boys both within and across the grades,
3. as a result of these observations, to generate, if possible, classes of problem-solving behaviors,
4. to study the performance of these boys with verbally-posed problems for any experiential effect.

Ten nonverbal problem-solving situations were devised which would require boys, in grades one, two and three, to become involved in the actions associated with division. Five of these situations were concerned with measurement division and five with partition division. Six boys from each of grades one, two and three, were selected from the pupils of an elementary school. Each boy in turn was presented with each of the ten problem situations, and was permitted to solve the problems in his own way. Their behaviors, as they solved the problems, were recorded on videotape.

After all the problem-solving behaviors of the eighteen boys had been recorded, two verbal tests were given to all of the children in the school. Each of these tests contained five measurement division problems and five partition division problems. The results obtained by a random sample of six boys from each grade were compared with the results obtained by the boys who participated in the nonverbal problem-solving study.

The analysis of the videotapes provided the data necessary for the first three purposes of this study. The data obtained from the verbal tests were used to test for any effect the experience of solving nonverbal problems might have on solving verbal problems of a similar kind.

II. CONCLUSIONS AND INTERPRETATIONS

1. The behaviors observed during the solution of these nonverbal problems were:

partitioning	verbalizing
counting	sighing
measuring	gestures
miming	fantasizing
observing	comparing
classifying	contemplating
calculating	

The most significant behaviors for solving nonverbal problems in division are partitioning, counting, measuring, classifying,

calculating and comparing. These were the behaviors which were generated by the problem situations themselves, and which were employed by the boys in order to arrive at solutions. The remainder of the behaviors, although stimulated by the problems, did not have a significant role in the development of the solutions.

It should be noted that those problems designed as measurement division did, in fact, result in measurement behaviors on the part of the boys. Similarly, the partition division situations tended to stimulate some kind of partitioning behavior. However, the boys in general did not show evidence of a systematic method of getting equal numbers of objects in a given number of sets.

There were numerous automatic behaviors such as moving the feet or scratching an arm that were not recorded. These general types of movements did not appear to be associated, in any way, with the solution of the problems.

Counting was by far the most common behavior observed. This is surprising when one considers that in partition problems, for example, counting is not required at all, except to find the number of objects in one of the sets. When this number is found, the number of objects in each of the other equivalent sets is immediately known. The occurrence of this behavior in situations where it is not necessary probably reflects the great emphasis on the process in early childhood. Most of the counting performed in the measurement problems was redundant. These problems could have been solved more efficiently if the child had recognised, without counting, the number in each of the equal groups. It would seem reasonable to suggest

that practice in group recognition without counting up to four or five, at least, might be a useful instructional objective in the early grades.

2. a. Partitioning does not appear to be a process that receives much attention in the early grades. Even the grade three boys appeared to have difficulty in getting the number of objects in each of the sets to be the same. The lack of development of any systematic procedure for this most useful process does not seem to be warranted.

   One only has to consider the important part that sharing plays in a young child's experience to know that partitioning can easily be taught. The child who systematically places one object in turn in each of the given sets and then repeats the procedure until all the objects have been distributed will be guaranteed that the number of objects in each of the sets will be the same, providing the division is exact. Such criterion behavior can be taught quite readily to children younger than those in this study (Nelson and Liedtke, 1972). The child who has been taught to partition in this way may want to count in order to check his work, but this would usually be done after the completion of the process.

   Procedures adopted by boys in this study to effect a partitioning varied considerably. Some tended to take several objects and place them in one of the given sets. They would then take another group of the same number and place these in the next set, and so on. The criterion used by some of these

boys appeared to be classification, if any existed. Sometimes these boys would exhaust the stock of objects before all the sets had been completed. On other occasions they had a surplus, and would then distribute the remaining objects among the sets in a random manner. Sometimes they guessed correctly the first time so that the sets all had the same number and there were none left over. More often, however, particularly with the younger boys, the procedure appeared to be a trial and error process, even though they understood what had to be found to solve the problem.

b. All but one of the subjects readily <u>used the materials and the problem-solving situations</u> to obtain solutions to the problems. If a major purpose of problem-solving in the early grades is to ensure that pupils understand what the problem is about, then it would appear that situations of the kind devised for this study have an important role to play. Although some of the boys could not effectively handle some of the processes required, to solve the problem, it was clear from the way they proceeded that the question asked in the problem was clearly understood. There was, however, some evidence that memory is a factor, for some boys became so involved with the manipulations of the materials that they forgot the question they were to answer.

c. <u>Measuring</u>, which in this study was defined as the process of forming subsets of a specified size from a larger set was readily carried out by most boys. However, most of them

had to count the number in the smaller sets to verify that there was the same number of objects in each. If a child had learned to recognise the number in small groups without counting, there would be a gain in efficiency when working with real objects. It has been noted previously that boys tended to use the measuring process even when partition would have been the preferred process. This, of course, is linked to the general lack of ability to partition with confidence.

d. There were two situations, one measurement and one partition, in which classifying appeared to be a reasonable procedure. (These were the situations in which animals were to be put in cages.) In the measurement case, simple classifying on the basis of type of animal led to a correct solution. In the partition case, however, simply classifying according to type of animal did not lead to a sensible solution. The 'noise' induced by this phenomenon clearly confused the majority of the subjects. This confusion was almost as apparent with the older boys as with the younger ones. There was considerable reluctance on the part of most boys to abandon the classes formation. For example, it did not appear correct to put bears and apes in the same cage. This same tendency to use a mathematically inappropriate but familiar procedure rather than attempt to devise a more appropriate new one has been noted.

e. Calculating did not appear to be a significant part of the regular mathematics program for the younger boys, understandably so for the grade one boys. The boys of grades one and two showed little attempt to use calculations in the

situations devised for this study, although there was evidence that grade three boys were using this process. This was probably a result of having been introduced to certain kinds of calculations in their mathematics program. One wonders whether it would have been used more extensively if paper and pencil had been provided and the boys told they could solve the problems on paper if they preferred. In any case, carrying out very complicated calculations mentally was probably avoided by most boys.

f. Miming, comparing, and observing were more often displayed by grade one and two boys. These are behaviors which may have been superfluous or redundant, and the tendency for the older boys to not exhibit them would appear to be some evidence of a development in the ability to select appropriate problem-solving behaviors.

g. No special attempt was made to encourage these children to verbalize their results, but the behavior was more common among the younger boys than the older boys. Verbalizations sometimes took the form of noises a boat or a car would make. In many cases there seemed to be a desire on the part of the boys to infuse as much realism into the situation as possible, even if it meant mimicing the sounds the objects would make if they were real.

h. Contemplating was a widely exhibited behavior, but appeared in only a few forms. The subjects would look away from the model on which they were working, and appear to be concentrating

on some aspect of the problem. It was easy for the observer to understand why Dunker (1945) and Gagné and Smith (1962) insisted on their subjects verbalizing as they solved their problems, because valuable insight into the subject's difficulties and strategies may be obtained. A study of the records of this investigation did not reveal any significant relation between the amount of verbalizing and the contemplating exhibited by individual boys.

3. It may not be too useful, but the behaviors generated by these problems could be broadly classified as:

   a. <u>Cognitive</u>: these would include behaviors associated with the mathematical processes as well as those indicative of thinking, for example, partitioning, counting, measuring, comparing, calculating, miming, observing, contemplating and classifying.

   b. <u>Affective</u>: sighing, fantasizing.

   c. <u>Motor</u>: gestures and verbalizing.

4. There were no significant differences in the verbal test results between the grade means of the 18 boys who had experienced the nonverbal problems, and the sample of boys from the grade who had no such experience. However the signs for five of the six comparisons, (two tests and three grades), were positive. The probability of this occurring by chance is $p < .11$ (Siegel, 1956, p. 250). It should be noted that no attempt was made to relate the actions and the manipulations in the nonverbal problems to calculations associated with solving verbal problems. Since

there is no strong evidence of a significant transfer one has to assume that the conditions would have to be altered if one were looking for such a transfer to occur. For example, one would want to know how much transfer would take place if specific steps were taken to relate the two kinds of problems.

With the exception of the grade one boys, all groups showed a slight but not significant increase in the mean scores obtained in the second verbal test. There was a significant difference ($p < .05$) between the mean scores for grade one and grade three, whereas the differences in means between grade one and grade two and between grade two and grade three were not statistically significant ($p > .20$), indicating that a difference in ages had some bearing on ability to solve verbal problems. In other words, the older the child, the greater his ability to solve the verbal problems.

III. PROBLEMS IN RESEARCH PROCEDURES

Although the pilot study provided the investigator with the opportunity to become familiar with the apparatus and the presentation of the problems to the children, several difficulties presented themselves during the study. These were partly technical in nature, partly due to individual differences, and partly accidental.

The videotape recorder and monitor performed satisfactorily during the pilot study. However, the initial recording at the school highlighted serious problems with this apparatus. These

concerned the most suitable placement of the microphone and the audio recording. The microphone was strapped to a table leg for two reasons, namely, to reduce any distraction caused by its presence on the table, and to prevent the microphone from being accidentally knocked off the table. The voice recording was found to be quite unsatisfactory with the microphone in this position, and it was eventually strapped semi-permanently to the table top with adhesive tape. Even so, some voices were so soft as to be barely audible at times.

The next problem was interference produced by the strong radio signal picked up from a local radio station, which at times almost completely drowned out the audio recording. This difficulty was overcome by replacing the V.T.R. with another similar unit which had been adapted to overcome this unwanted reception.

A further difficulty was due to the need for economy with the videotapes, and the uncertainty about the time required by any one boy to solve a problem. On two occasions the tape "ran out" in the last stages of a solution. When this occurred, care was taken to make a written record of proceedings. However, it would have been better if the tape had been replaced before the solving of a problem commenced if the reel appeared to be almost used up. It was found that a one-hour tape was quite adequate to record the activities of any one boy.

As was reported earlier, the investigation was carried out in the school science room. There were several occasions when interruptions occurred due to children entering the room to obtain

materials or equipment. An attempt was made to reduce these to a minimum by seeking the teachers' cooperation, which was willingly given.

Another difficulty experienced concerned the consistency of oral presentation. Although care was taken to present the problem in the way it was set out on the card, there were many instances where boys asked questions or made comments; this usually resulted in the complete problem being stated again after the query had been resolved. This meant that other boys who, although likewise puzzled, did not speak up and have the point clarified, were at a disadvantage. Sometimes the comments would indicate that the subject was not comprehending the problem, or was attaching importance to aspects of the physical situation which had no bearing on the solution to the problem. Correcting these may have given these particular subjects an advantage over those boys who were not so vocal.

A problem which faced the investigator was concerned with the frustration shown by a third grade student. The approach taken by this boy to problems II and VI, both partitioning problems, resulted in a mental set which obstructed his obtaining a satisfactory solution. He was obviously frustrated as was indicated by sighing and verbalizing. The dilemma was whether or not to make a suggestion which may have caused him to approach the problem differently. The investigator was concerned that the boy's attitude to further problems may have been affected by his obvious trouble with the problems. After a period of five minutes had elapsed, during which the student came no closer to obtaining a solution, the investigator drew his attention to the

concept employed in a previous problem. This immediately resolved the problem, for the student forsook his previous unfruitful efforts, and approached the problem from the correct point of view.

There was also the instance in which a grade one boy showed no interest in problem II. He placed posts in holes and then added poles. His actions did not lead to a solution to the problem. The investigator briefly questioned the student to ensure that he understood what was required. He was able to give a fluent statement of the problem, but was apparently no longer interested in it.

IV. SUGGESTIONS FOR IMPROVEMENT OF RESEARCH PROCEDURES

a. When an analysis of behaviors in this study was carried out no attempt was made to keep an accurate record of the frequency of occurrence of each behavior for each problem. In retrospect it seems that this would be a useful statistic to possess, as it could be an indication of a student's thinking about the problem, and a measure of the capacity of a problem to generate behaviors.

b. A better understanding of a student's thinking about a problem may be gained by a close study of the sequence of behaviors. What specific behavior pattern leads to a quick solution to a problem could prove to be worthwhile knowledge. Is a particular behavior a necessary prelude to obtaining a solution? Which behavior pattern is indicative of a student's understanding of the problem? These questions suggest that a time-and-motion study of a student's behaviors in a nonverbal problem solving situation could result in a

better understanding of children's thought processes when solving problems.

c. It would appear that a smaller number of problems would generate the same variety of behaviors. This would prove advantageous from a number of points of view.

i. It would allow for a closer inspection of the behavior patterns for those problems used in the study.

ii. It would reduce any pupil fatigue factor, which may have been present in the present study, although it was not so obvious as to cause concern.

d. A tighter control should be kept of the environment in which the study is made, to reduce the number of possible distractions. Although the use of a recording studio has certain logistic problems associated with it, the control which it gives the investigator over the external conditions of the study seems to be a desirable factor.

V. IMPLICATIONS FOR THE CLASSROOM TEACHER

The results of the study show that partitive division is not taught in grades 1, 2 or 3. The basic concepts of partitive division are rooted in simple, day by day occurrences in an elementary school, and, if not treated in terms of materials from the environment, may result in difficulties in comprehension when the abstract situation is broached.

It appears to the investigator that the classroom teacher

could develop situations to foster:

1. the "fairshares" concept, emphasising the sharing aspect. This could be developed through the distribution of candy and through the use of colored paper in art work.
2. motivation and interest. The situations call for role playing in physical situations into which division and other mathematical relations have been structured.
3. group counting, first with objects, later with numbers.
4. more practice with one to one correspondence. Because some second and third grade boys were unable to count correctly, it appears that teachers of these grades could provide more practice to avoid possible deficiencies.

VI. IMPLICATIONS FOR FURTHER RESEARCH

1. The present study involved a relatively small sample, a single investigator and some technical equipment. Because of these factors several difficulties occurred which have been already recorded. To overcome these and to establish a large body of knowledge on how children solve nonverbal problems, much more research needs to be carried out. This would involve a team of investigators accompanied by support staff of designers and technicians. In this way findings of a substantial nature could occur.
2. The student sample used in this study was drawn from a population which was fairly homogeneous with respect to socio-economic status. To test whether the behaviors generated are characteristic

of the total student population, the study should be extended to include boys from other socio-economic backgrounds.

3. This study was concerned with the behaviors of boys in nonverbal problem-solving situations. Informal observations of girls in similar situations suggest that this study should be replicated to include children of both sexes. This could result in a wider spectrum of behaviors being generated.
4. An investigation of the time-sequence of behaviors with individual children would appear to be desirable. Recording procedures similar to those used in this study would be appropriate, but the method of analysis would need modification. A time-motion case study technique would appear to be most appropriate for this kind of research.
5. Furthermore, the effect of experience with nonverbal problem-solving on developing an ability to solve verbal problems has not been fully investigated. Studies in which teaching procedures are aimed at achieving this transfer are deserving of consideration.

VIII. CONCLUSION

Brownell (1942), in discussing theoretical and practical aspects of problem solving, listed some suggestions for developing ability in problem solving.

> Part of real expertness in problem solving is the ability to differentiate between the reasonable and the absurd, the logical and the illogical. Instead of being 'protected' from error, the child

> should many times be exposed to error and be encouraged to detect and to demonstrate what is wrong, and why (p. 440).

He then added:

> A problem solving attitude, an inquiring and questioning mind, is a desirable educational outcome, and it is possible of development. . . . The attitude is produced by continued experience in solving real problems, one consequence of which is that the learner comes to expect new problems and to look for them (p. 440).

The investigator maintains that the type of problem-solving described in this study can be of real practical benefit to a teacher who wishes to develop this problem-solving expertise and attitude in children in grades one to three, and it can provide the means for bringing real-world problems into the classroom. Non-verbal problem-solving activities such as those described in this study appear, to the investigator, to provide a means by which the elementary school teacher can develop the attitudes and abilities referred to by Brownell (1942).

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APPENDIX A

VERBAL PROBLEM TEST 1

1. If you have 16 toy train cars, how many trains with four cars can you make?

2. If you have 15 toy train cars and three engines, and you make trains all the same size, how many cars do you put after each engine?

3. Twelve children are waiting to ride paddle boats on the lake at Mayfair Park. Each paddle boat carries two children. How many boats are needed?

4. The train at Storyland Zoo has six cars. If 24 children are waiting for a ride and the driver puts the same number of children in each car, how many people will ride in each car?

5. If a teacher takes her class of 20 children by car on a picnic, and her car can carry five children, how many trips will she make to get all the class to the picnic?

6. If five mothers volunteer to drive the 20 children to the picnic, and each car carries the same number of children, how many children will go in each car?

7. If I give 18 marbles to my three boys to share equally, how many will each get?

8. If your mother washes 14 socks, how many pairs of socks are in

the wash?

9. How many sets of four can you make from 24?

10. If you make 12 into three equal sets, how many are in each set?

VERBAL PROBLEM TEST 2

1. If 16 children fill up two rows in a classroom, how many seats are in each row?

2. Suppose you have 18 chairs and you put six chairs in a row. How many rows can you make?

3. Four children each brought the same number of records to a party. If there were 12 records altogether, how many did each bring?

4. There are 15 children waiting for a ride on the train at the Game Farm. If five children are put in each car, how many cars will be filled?

5. A father buys 15 tickets at the Klondike Carnival, and he has three children. How many tickets does he give each child?

6. If your mother has 24 candies and she puts six in each party bag, how many party bags does she fill?

7. How many pairs of shoes can you make from a pile of 22 shoes?

8. If a class of 12 children go on a canoe trip, and four children fit in each canoe, how many canoes will be needed?

9. How many sets of 3 are there in 21?

10. If 25 is divided into 5 equal sets, how many will there be in each set?

APPENDIX B

APPENDIX B

The following equipment was provided by the Audiovisual Media Centre of the Faculty of Education, University of Alberta, for use in this study.

a. Javelin half-inch New Format Video Tape Recorder VTR 200

b. Sony half-inch New Format VTR AV 3600

c. Sony Television Movie Camera, Model AVC 3200

d. Electrohome Monitor Receiver ETV 6

e. Sony Microphone F-98.

APPENDIX C

TABLE 25

COMPARISON OF MEANS OF SUBJECTS IN

GRADES ONE, TWO, AND THREE

TEST ONE

Grade	Number	Mean	Variance	S. Deviation
1	6	2.8333	11.7667	3.4303
2	6	5.8333	4.1667	2.0412
3	6	7.3333	6.6667	2.5820
Total	18	5.3333	9.7778	3.1269

Analysis of Variance

Source	SS	MS	DF	F	P
Groups	62.9	31.50	2	4.18	0.0360
Error	113.00	7.53	15		

TEST TWO

Grade	Number	Mean	Variance	S. Deviation
1	6	2.0000	2.0000	1.4142
2	6	7.5000	12.3000	3.5071
3	6	8.1667	0.9667	0.9832
Total	18	5.8889	11.8765	3.4462

Analysis of Variance

Source	SS	MS	DF	F	P
Groups	137.44	68.72	2	13.5	0.0004
Error	76.33	5.09	15		

TABLE 26

COMPARISON OF MEANS OF SUBJECTS AND

GRADE ONE RANDOM SAMPLE

TEST ONE

Group	Number	Mean	Variance	S. Deviation
1	6	2.8333	11.7667	3.4303
2	6	2.0000	1.2000	1.0954
Total	12	2.4167	5.5764	2.3614

Analysis of Variance

Source	SS	MS	DF	F	P
Groups	2.0833	2.08	1	0.32	0.5833
Error	64.8333	6.48	10		

TEST TWO

Group	Number	Mean	Variance	S. Deviation
1	6	2.0000	2.0000	1.4142
2	6	2.5000	3.1000	1.7607
Total	12	2.2500	2.1875	1.4790

Analysis of Variance

Source	SS	MS	DF	F	P
Groups	.7500	0.75	1	0.29	0.5994
Error	25.5000	2.55	10		

TABLE 27

COMPARISON OF MEANS OF SUBJECTS AND

GRADE TWO RANDOM SAMPLE

TEST ONE

Group	Number	Mean	Variance	S. Deviation
1	6	5.8333	4.1667	2.0412
2	6	5.6667	4.2667	2.0656
Total	12	5.7500	3.5208	1.8764

Analysis of Variance

Source	SS	MS	DF	F	P
Groups	8.325	0.08	1	0.02	0.8910
Error	42.17	4.22	10		

TEST TWO

Group	Number	Mean	Variance	S. Deviation
1	6	7.5000	12.3000	3.5071
2	6	6.5000	3.9000	1.9748
Total	12	7.0000	7.0000	2.6458

Analysis of Variance

Source	SS	MS	DF	F	P
Groups	3.000	3.00	1	0.37	0.5564
Error	81.000	8.10	10		

TABLE 28

COMPARISON OF MEANS OF SUBJECTS AND

GRADE THREE RANDOM SAMPLE

TEST ONE

Group	Number	Mean	Variance	S. Deviation
1	6	7.3333	6.6667	2.5820
2	6	7.0000	14.4000	3.7947
Total	12	7.1667	8.8056	2.9674

Analysis of Variance

Source	SS	MS	DF	F	P
Groups	0.3333	0.33	1	0.03	0.8624
Error	105.3333	10.53	10		

TEST TWO

Group	Number	Mean	Variance	S. Deviation
1	6	8.1667	0.9667	0.9832
2	6	7.3333	9.4667	3.0768
Total	12	7.7500	4.5208	2.1262

Analysis of Variance

Source	SS	MS	DF	F	P
Groups	2.0830	2.08	1	0.40	0.5416
Error	52.1670	5.22	10		